Identifying Contagion

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Abstract

Identifying contagion effects during periods of financial crisis is known to be complicated by the changing volatility of asset returns during periods of stress. To untangle this we propose a GARCH common features approach, where systemic risk emerges from a common factor source (or indeed multiple factor sources) with contagion evident through possible changes in the factor loadings relating to the common factor(s). Within a portfolio mimicking factor framework this can be identified using moment conditions. We use this framework to identify contagion in three illustrations involving both single and multiple factor specifications; to the Asian currency markets in 1997-98, to US sectoral equity indices in 2007-2009 and to the CDS market during the European sovereign debt crisis of 2010-2013. The results reveal the extent to which contagion effects may be masked by not accounting for the sources of changed volatility apparent in simple measures such as correlation.

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1 Introduction

There is widespread agreement that the challenge for identifying contagion is to disentangle it from interdependence. In particular, contagion is not simply revealed by increased correlation of performance indicators during a crisis period – rising correlation coefficients are polluted by the rising volatility conditions which are almost invariably associated with periods of financial stress. Forbes and Rigobon (2002) propose a neat way of correcting the correlation coefficient in order to catch only that part which is not due to rising volatility. However, when contemplating the potential for contagion from a 'source' market to a 'target' market, it is clear that the Forbes and Rigobon correction does not take account of the fact that the volatility of the target market may change both for reasons associated with the source, and for reasons of its own. If this idiosyncratic volatility in the target market is higher (lower) in crisis times than in non-crisis times, we argue that the Forbes and Rigobon correction overestimates (underestimates) the spurious component of the correlation increase.

These observations suggest a factor model structure when considering contagion from multiple sources towards multiple potential targets. It is the simultaneous volatility increase due to the rising volatility of common factors that pertains to interdependence, and should not be dubbed contagion. This framework is compatible with the view of contagion as correlation in excess of that expected via fundamentals, as in Bekaert et al (2005), or later, and more directly related, in Bekaert et al (2014) as "the comovement in excess of that implied by the factor model". While we concur with this definition, our empirical strategy differs.

The originality of our approach is to let the data speak by basing identification of the fundamentals and the link to asset correlation on a statistical testing strategy rather than on an a priori definition of economic fundamentals. Following the seminal work of King et al (1994) we note first, that the key to identifying changing conditions for portfolio diversification is to see the time-varying volatility of returns as "induced by changes in the underlying factors", and second, that assuming time-varying conditional variance may facilitate identification of the factor model.

On one hand, like King et al (1994), we set the focus on factors whose role is to capture the time variations of volatility that are predictable – in contrast with unpredictable structural breaks in crisis times. We focus on the unpredictable part of correlation as characterized by idiosyncratic terms, and for these to play a role in contagion via structural changes during crisis periods. In contrast with King et al (1994), our factor model allows idiosyncratic risks to be correlated across assets. For us 'idiosyncratic' means uncorrelated with the factors which drive predictable (GARCH-type for example) components of return volatilities. One of the two channels of contagion we identify will be through structural breaks in so-called 'idiosyncratic betas' (betas of target on source induced only by their idiosyncratic components). Idiosyncratic risks are by definition time invariant, up to structural changes in the crisis period, and we will identify factors as capturing all time-varying components of return volatilities. Although we never resort to any specific GARCH or stochastic volatility dynamics, our multivariate volatility model with latent factors can be seen as inspired either by GARCH-factor models or common factor stochastic volatility models studied in a maximum likelihood framework by Diebold and Nerlove (1989) and Engle et al (1990) for GARCH factors as well as Fiorentini et al (2004) for stochastic volatility factors. Note that all of these models are more restrictive than ours as they are both fully parametric (in a likelihood setting) and preclude correlation between idiosyncratic risks. Our model is semi-parametric in nature and implemented with a GMM approach.

On the other hand, like Bekaert et al (2014), we want to characterize contagion as comovements which are not explained by the factor model. Although our statistical strategies are different from Bekaert et al (2014), our definitions of contagion are essentially identical. With weekly data, Bekaert et al (2014) run regressions of returns on lagged values and contemporaneous factors (three value-weighted market indices) with time-varying beta coefficients. Contagion is identified by a non-zero coefficient for a crisis dummy either in time-varying betas or in the intercept. In our framework betas are constant by definition (since time-varying volatilities are fully captured by factors), except for structural breaks at times of crisis. Since we work with daily data, we focus on conditional variances and consider structural changes in idiosyncratic betas rather than intercept shifts. However, our two preferred characterizations of contagion as structural changes in either conditional factor loadings or in idiosyncratic betas match the spirit of the two dummy coefficients on conditional factor loadings and intercepts in Bekaert et al (2014).

Our semi-parametric framework for contagion identification fits within the "GARCH common features" framework as put forward by Engle and Kozicki (1993) and revisited for statistical inference by Doz and Renault (2006) and Dovonon and Renault (2013). The key idea is that heteroskedasticity goes through only a few latent common factors while many linear combinations of primitive returns (called common features and interpreted as returns on portfolios) are actually homoskedastic, a feature implicitly incorporated in the contagion model of Dungey and Martin (2007). The common factors are the source of potential contagion (the systematic risk), where contagion is defined as the change in the factor loading of each of the n primitive asset returns (the n potential targets) on the systematic risk. [1]

Our framework cannot avoid the reflection problem of Manski (1993): we need some prior information to specify the reference assets. In other words, we must pick a few assets that are not in our target set and consider them as the "mimicking portfolio"; that is the portfolios whose returns convey the information about the underlying systematic risk. Contagion will then be characterized by changes in factor loadings of the n assets on the latent common factors. As it is common in the existing literature to exogenously choose the 'source' assets for contagion based on observed events this does not present a disadvantage.

To protect ourselves from the spurious identification of contagion, highlighted in Billio and Pelizzon (2003), we ensure that the increase in volatility is due at least as much to the idiosyncratic component as to the potential source of contagion through systematic risk. We recognize that the share of the observed portfolio variance that is borne by the common factors is somewhat arbitrary (above some lower bound in order to capture the genuine time-varying part), although robustness tests suggest there is little sensitivity to this choice.

Whilst contagion is of considerable importance to policy makers and investors alike, there is significant disagreement in the existing literature on how to detect its presence, as the following examples illustrate. Identification via heteroskedasticity

[1] We are very close in spirit, albeit in a completely different framework, to some work developed simultaneously and independently by Darolles et al (2015) who attempt to disentangle frailty (the latent factor explaining correlation in default occurrences) from contagion; see also Duffie et al (2009).

forms the basis of tests in Bekaert et al (2014), Dungey et al (2010), Dungey and Martin (2007), and Corsetti et al (2005), with alternative means of controlling for evolution in conditional correlation in Caporale et al (2005) and Kasch and Caporin (2013) who incorporate DCC models, and in Markov switching frameworks such as Ajay et al (2013). Relationships between tail events are differentiated from those in 'normal' times in papers using co-exceedance measures such as Bae et al (2003), Boyson et al (2010), quantiles in Baur and Schulze (2005), Caporin et al (2014) and copulas in Busetti and Harvey (2011) and Rodriguez (2007). The impact of extreme events such as outliers or jumps represents contagion effects in Favero and Giavazzi (2002) and Aït-Sahalia et al (2014), while papers such as Longstaff (2010) rely on changes in transmission mechanisms across periods without specifically accounting for changes in underlying volatility. However, the common feature of all of these models is a concern with the change in the loading on the transmission of the source to the target asset during a crisis period.

We estimate contagion effects as a change in the loading on the source factors for three examples covering different asset markets in different geographical regions and different crisis periods – currencies during the 1997-1998 Asian crisis, sectors of the US equity market in 2007-2009 and European sovereign CDS spreads for 2008-2013. These allow us to demonstrate both single and multiple factor specifications and the reflection problem. Contagion effects are sometimes the result of increased loadings on the source factors, but equivalently sometimes reveal decreased loadings as assets disconnect from the crisis source. These effects are not necessarily revealed in analyses that concentrate on changes in correlation or (unconditional) regression coefficients, as the increased residual volatility for the asset may be sufficient to mask the change in the underlying factor loadings.

The paper proceeds as follows. Section 2 sets up the problem of identifying contagion as separate from interdependence through carefully differentiating explained and unexplained volatility and predicted and unpredicted volatility for the target asset in order to develop our modelling framework. Section 3 explains the econometric method and testing strategies adopted in Section 4, where three empirical examples are given. Section 5 concludes.

2 Interdependence versus contagion

In this section we analyze the different reasons why comparing the correlation between the returns of two assets during a crisis period and the corresponding correlation during a non-crisis period is not an accurate way to identify contagion.

2.1 Source versus target

We are mainly interested in the impact of a given source of contagion, namely some reference asset return r_0, on a family of possible targets, namely asset returns $r_i, i = 1, ..., n$. It is then natural to think about the relationship between source and target in a linear regression framework:

$$r_i = \alpha_i + \beta_i r_0 + u_i, \quad E[u_i] = 0, Cov[r_0, u_i] = 0$$

The contagion effect will then be identified as a structural break in the joint distribution of asset returns. Since financial crises correspond in general to a pervasive increase of volatility among all asset returns, we will actually consider two sets of regression equations, one for the crisis period (defined by an index H for High volatility) and one for the non-crisis period (designated by an index L for Low volatility):[2]

$$r_{iH} = \alpha_{iH} + \beta_{iH} r_{0H} + u_{iH}, \quad E_H[u_{iH}] = 0, Cov_H[r_{0H}, u_{iH}] = 0 \tag{1}$$

$$r_{iL} = \alpha_{iL} + \beta_{iL} r_{0L} + u_{iL}, \quad E_L[u_{iL}] = 0, Cov_L[r_{0L}, u_{iL}] = 0 \tag{2}$$

With obvious notations:

$$\beta_{iL} = \rho_{iL} \frac{\sigma_{iL}}{\sigma_{0L}}, \beta_{iH} = \rho_{iH} \frac{\sigma_{iH}}{\sigma_{0H}} \tag{3}$$

As it is now well known, formula (3) clearly shows why comparing correlations ρ_{iH} and ρ_{iL} to identify contagion is biased towards finding contagion if the rate of increase of volatility $(\sigma_{0H}/\sigma_{0L})$ for the source country exceeds the rate of increase $(\sigma_{iH}/\sigma_{iL})$ for the target country. More precisely, when $\beta_{iH} = \beta_{iL}$, we have:

[2] In this way we deviate importantly from Bekaert et al (2014), Forbes and Rigobon (2002) and the regression model approach of Dungey et al (2005) who each impose a single homoskedastic error structure across both the non-crisis and crisis periods.

$$\rho_{iH} > \rho_{iL} \Leftrightarrow \frac{\sigma_{iH}}{\sigma_{iL}} < \frac{\sigma_{0H}}{\sigma_{0L}}$$

This remark led Forbes and Rigobon (2002) to propose a corrected correlation measure for the purpose of contagion identification, based on the rates of increase of volatility in both markets. We will now show that such a correction is not sufficient in general.

2.2 Unexplained versus explained volatility

By standard decomposition of variance formulas, we define unexplained volatility in both crisis and non-crisis periods as follows:

$$\begin{aligned} \omega_{iH} &= Var_H(u_{iH}) = (1-\rho_{iH}^2)\sigma_{iH}^2 \\ \omega_{iL} &= Var_L(u_{iL}) = (1-\rho_{iL}^2)\sigma_{iL}^2 \end{aligned}$$

We can then prove the following identity:

Proposition 2.1.:

$$\beta_{iH} = \beta_{iL} \Rightarrow \rho_{iH} = \rho_{iL} \left[\frac{1+\delta}{1+\delta\rho_{iL}^2 + \frac{\omega_{iH}-\omega_{iL}}{\sigma_{iL}^2}} \right]^{1/2}$$

where:

$$1+\delta = \frac{\sigma_{0H}^2}{\sigma_{0L}^2}$$

Proof:

$$\begin{aligned} \beta_{iH} &= \beta_{iL} \Leftrightarrow \rho_{iH}\frac{\sigma_{iH}}{\sigma_{0H}} = \rho_{iL}\frac{\sigma_{iL}}{\sigma_{0L}} \\ &\Leftrightarrow \rho_{iH} = \rho_{iL}[1+\delta]^{1/2}\frac{\sigma_{iL}}{\sigma_{iH}} \end{aligned}$$

But with $\beta = \beta_{iH} = \beta_{iL}$:

$$\frac{\sigma_{iH}^2}{\sigma_{iL}^2} = \sigma_{iH}^2\frac{\rho_{iL}^2}{\beta^2\sigma_{0L}^2} = \rho_{iL}^2\frac{\beta^2\sigma_{0H}^2+\omega_{iH}}{\beta^2\sigma_{0L}^2} = \rho_{iL}^2\left[1+\delta+\frac{\omega_{iH}}{\beta^2\sigma_{0L}^2}\right]$$

Therefore we will get the announced result if we show that:

$$\rho_{iL}^2 + \rho_{iL}^2\frac{\omega_{iH}}{\beta^2\sigma_{0L}^2} = 1 + \frac{\omega_{iH}-\omega_{iL}}{\sigma_{iL}^2}$$

that is:

$$\rho_{iL}^2 \sigma_{iL}^2 + \omega_{iH} = \sigma_{iL}^2 + \omega_{iH} - \omega_{iL}$$

which is true since $\omega_{iL} = (1 - \rho_{iL}^2)\sigma_{iL}^2$.

QED

Forbes and Rigobon (2002) set the focus on the increasing function (for positive ρ):

$$\phi(\rho) = \rho \left[\frac{1+\delta}{1+\delta\rho^2} \right]^{1/2}$$

They propose to correct ρ_{iH} as follows:

$$\tilde{\rho}_{iH} = \phi^{-1}(\rho_{iH}) \tag{4}$$

before assessing contagion through correlation increase. They argue that the relevant test for contagion is indeed the test of the inequality $\tilde{\rho}_{iH} > \rho_{iL}$. Their rationale is as follows: If the contagion were spurious (in the sense that indeed $\beta_{iH} = \beta_{iL}$), we would have $\rho_{iH} = \phi(\rho_{iL})$ and thus $\tilde{\rho}_{iH} = \rho_{iL}$. Thus, computing the corrected correlation is an edge against the upper-bias of ρ_{iH} that may spuriously lead to the conclusion that there was a contagion effect, while it was only an artefact of soaring volatility ($\delta > 0$). Our general proposition above shows that their argument is fully correct only in the particular case $\omega_{iH} = \omega_{iL}$, while by contrast (still assuming $\beta_{iH} = \beta_{iL}$):

$$\omega_{iH} > \omega_{iL} \Rightarrow \rho_{iH} < \phi(\rho_{iL}) \Rightarrow \tilde{\rho}_{iH} < \rho_{iL}$$

In this case, as pointed out by Billio and Pelizzon (2003), Forbes and Rigobon (2002) overestimate the spurious component of correlation increase (and thus overcorrect for it) because "a significant part of the increase in volatility is due to the idiosyncratic component". However, we must acknowledge that there is no such thing as a generalization of Forbes and Rigobon (2002) bias correction strategy for the general case where idiosyncratic risk may change. There is no alternative to the function $\phi(.)$ above since the correction term $\frac{\omega_{iH} - \omega_{iL}}{\sigma_{iL}^2}$ in Proposition 2.1. is not a function of ρ_{iL} alone.

The bottomline is that only variation of beta coefficients between non-crisis and crisis periods (corresponding respectively to equations (2) and (1)) provide some reliable measures of contagion.

2.3 Predictable versus unpredictable volatility

In a time of crisis, all the parameters of the economy may change. However, when trying to identify contagion, we are more interested in knowing whether change is about the structural linkages in the economy. In particular, since it is known that asset return volatility is stochastically time-varying and highly persistent, an important aspect of contagion would be a change in regime in these aspects. With respect to the approach of Forbes and Rigobon (2002) described in previous sections, we will introduce two additional features.

First, we set the focus on volatility dynamics rather than on some otherwise constant volatility parameters that suffer from structural breaks in the crisis period. Second, the source of contagion is no longer a given asset return but rather one or more latent factor(s) that are responsible for time variations of volatility. For identification purposes, the dynamics of these latent factors will be studied through the observed dynamics of some mimicking portfolios.

For the sake of expositional simplicity, let us depict the factors, f_i, as responsible for volatility dynamics while the so-called source returns, r_{0j}, are actually the return of some mimicking portfolios. In all subsequent equations, we refer to a given increasing filtration $(I_t), t = 1, 2..., T$, such that the first two conditional moments of returns observed a time $(t+1)$ are computed given the information I_t available at time t, and then accordingly denoted by $E_t(.), Var_t(.)$ and $Cov_t(.)$.

2.4 Model for the source returns

Consider a potential of $k < n$ source returns as noisy observations of the latent volatility factors; to conserve space we portray this in the $k = 2$ case here, but the generic k factor case is easily generalizable:

$$\begin{bmatrix} r_{01,t+1} \\ r_{02,t+1} \end{bmatrix} = \begin{bmatrix} f_{1,t+1} \\ f_{2,t+1} \end{bmatrix} + \begin{bmatrix} u_{01,t+1} \\ u_{02,t+1} \end{bmatrix} = F_{t+1} + U_{0,t+1}$$

where $U_{0,t+1} = [u_{01,t+1}, u_{02,t+1}]'$ contains homoskedastic error terms, and the factors are conditionally heteroskedastic:

$$E_t\left(U_{0,t+1}\right) = 0, Var_t(U_{0,t+1}) = \Omega_{00}$$

$$E_t\left(f_{t+1}\right)=0, Var_t(F_{t+1})=\left[\begin{array}{cc}\sigma_{11,t}^2 & \sigma_{12,t}\\ \sigma_{12,t} & \sigma_{22,t}^2\end{array}\right]$$

For expositional simplicity we assume that all returns and factors have a zero-conditional expectation given past information. Assuming zero conditional co-variance between factors and noise, the variance of the source is decomposed into time-varying and constant components.

$$Var_t\left(\left[\begin{array}{c} r_{01,t+1}\\ r_{02,t+1}\end{array}\right]\right)=\Sigma_{0,t}=Var_t(F_{t+1})+\Omega_{00}$$

Since the conditional covariances of the two source returns may be time-varying, it may not be possible to normalize the two latent factors to make then conditionally uncorrelated. By contrast, we can assume without loss of generality that they are unconditionally uncorrelated, that is:

$$E\left[Var_t(F_{t+1})\right]=\left[\begin{array}{cc}\sigma^2(f_1) & 0\\ 0 & \sigma^2(f_2)\end{array}\right]$$

Note that the unconditional variance of factors is not identified. The time-varying part of return variance can always be artificially inflated by incorporating a constant component. In other words, it takes an identification assumption to decide the share of the variance of the sources carried by the factors:

Identification assumption $A(\alpha)$

For some $\alpha_j \in]0,1[$ given:

$$\frac{\sigma^2(f_j)}{Var(r_{0j,t+1})}=\alpha_j$$

Note that for $j=1,2$ the parameters α_j can be interpreted as the (unconditional) squared correlation coefficient between the volatility factor $f_{j,t+1}$ and its mimicking portfolio return $r_{0j,t+1}$. By choice of α_j, we identify factors that are more correlated with the source when α_j is large. The smaller the α_j, the smaller the part of time-invariant volatility carried by the factor, since for the source, the residual variance is:

$$\Omega_{00}=\left[\begin{array}{cc}\omega_{00,11} & \omega_{00,12}\\ \omega_{00,21} & \omega_{00,22}\end{array}\right]=\left[\begin{array}{cc}(1-\alpha_1)Var(r_{01,t+1}) & \omega_{00,12}\\ \omega_{00,21} & (1-\alpha_2)Var(r_{02,t+1})\end{array}\right]$$

The choice of α_j is constrained by the fact that the volatility of the factor must be sufficiently high to allow the factor to capture at least the time-varying part of the variance of the source. That is the residual variance for each source asset is bounded as follows:

$$\omega_{00,jj} \leq \min_{1\leq t\leq T} \left[Var_t\left(r_{0j,t+1}\right)\right]$$

Then, the variance of the factor would be kept at its minimum possible value if one chooses $\alpha_j = \bar{\alpha}_j$ defined as follows:

$$\bar{\alpha}_j = 1 - \frac{\min_{1\leq t\leq T}\left[Var_t\left(r_{0j,t+1}\right)\right]}{Var(r_{0j,t+1})}.$$

The value of $\bar{\alpha}_j$ increases as the share of conditional returns on unconditional returns decreases. Our choice will be to take the same value of α_j for $A(\alpha)$ for all source assets α_j. Since we want to take the minimum possible value, we take $\alpha_j = \overline{\alpha} = \max(\overline{\alpha}_j)$. Empirically, the assumption of $\alpha_j = \bar{\alpha}$ is innocuous in our examples, as long as the minimum condition is respected the results are relatively insensitive to the choice of value for α_j.

2.5 Model for the target return

The key idea is for the volatility factor to capture the time-varying volatility. The model entails two restrictions: First, target returns $r_{i,t+1}$, have a time invariant conditional regression coefficient b_i on the volatility factor. Second, the vectors of residuals of this regression are homoskedastic. For brevity consider the case of $(n-k)$ where $k=2$ target returns, admitting that two of the initial n target returns are now seen as sources.

Formally, for $i = 1, ..., n-2 : j = 1, 2$

$$\begin{aligned} r_{i,t+1} &= b_{i1} f_{1,t+1} + b_{i2} f_{2,t+1} + u_{i,t+1} \\ E_t\left(u_{i,t+1}\right) &= 0, Cov_t[u_{i,t+1}, u_{j,t+1}] = \omega_{ij} \\ Cov_t[f_{1,t+1}, u_{i,t+1}] &= Cov_t[f_{2,t+1}, u_{i,t+1}] = 0 \end{aligned} \tag{5}$$

and for $i = 1, ..., n-2 : j = 1, 2$

$$Cov_t[u_{i,t+1}, u_{0j,t+1}] = \omega_{i,0j},$$

Then the two factor model (5), jointly with the specification of the shares α_i for factor volatility, provides a decomposition of unconditional beta coefficients of asset $i = 1, ..., n-1$ defined as:

$$\beta_{ij} = \frac{Cov[r_{i,t+1}, r_{0j,t+1}]}{Var(r_{0j,t+1})}, j = 1, 2$$

Since by definition:

$$\begin{aligned} Cov_t[r_{i,t+1}, r_{01,t+1}] &= b_{i1}\sigma^2_{11,t} + b_{i2}\sigma^2_{12,t} + \omega_{i,01} \\ Cov_t[r_{i,t+1}, r_{02,t+1}] &= b_{i1}\sigma^2_{12,t} + b_{i2}\sigma^2_{22,t} + \omega_{i,02} \end{aligned}$$

by taking unconditional expectations:

$$\begin{aligned} Cov[r_{i,t+1}, r_{01,t+1}] &= b_{i1}\sigma^2(f_1) + \omega_{i,01} \\ Cov[r_{i,t+1}, r_{02,t+1}] &= b_{i2}\sigma^2(f_2) + \omega_{i,02} \end{aligned}$$

and dividing on both sides respectively by $Var(r_{01,t+1}) = \frac{\sigma^2(f_1)}{\alpha_1} = \frac{\omega_{00,11}}{1-\alpha_1}$ and by $Var(r_{02,t+1}) = \frac{\sigma^2(f_2)}{\alpha_2} = \frac{\omega_{00,22}}{1-\alpha_2}$ we get:

$$\begin{aligned} \beta_{i1} &= \alpha_1 b_{i1} + (1-\alpha_1)\gamma_{i1} \\ \beta_{i2} &= \alpha_2 b_{i2} + (1-\alpha_2)\gamma_{i2} \end{aligned} \quad (6)$$

where $\gamma_{ij} = \omega_{i,0j}/\omega_{00,jj}, j = 1, 2$ is the regression coefficient (both a conditional and an unconditional one) of u_i on u_{0j}. As rigorously explained in the next section, under the maintained assumption (5), the four beta coefficients β_{ij} and $b_{ij}, j = 1, 2$, are identified from the observation of the time series $(r_{i,t})_{1\leq t\leq T}$, $i = 0, 1...., n$ of asset returns. By contrast, identification of $\gamma_{ij}, j = 1, 2$, takes into account the choice of the values α_j of the share of the variance of the factors in the total variance of the source returns.

Our identification strategy will then be germane to the "identification from change in variance" approach promoted by Rigobon (2003). We will exogenously pick a value of α_j and keep it invariant from the non-crisis period to crisis period. Note that this is a natural normalization condition in these applications – the mimicking portfolio keeps the same correlation with the latent volatility factor. This allows us to find values of β_{ij}, b_{ij} and γ_{ij} in both low volatility and high volatility regimes. We are able to measure the structural change of the unconditional regression coefficient, β_{ij} (of the target on the source), but also to disentangle the two components of this change as the structural change in the impact of the volatility factor on the different markets via b_{ij}, and the structural changes in the time invariant residual correlations between markets, γ_{ij}. As the volatility factor is highly predictable then the first type of contagion, changes in b_{ij} are of greater interest as far as economic policy is concerned. In other words, we see that the (structural changes in) unconditional β_{ij} coefficients are imprecise signals of contagion since the phenomena of interest (structural changes in b_{ij}) may be blurred by idiosyncratic issues.

3 Econometric Inference

For sake of notational simplicity, it is convenient to always consider that we have a set of $(n+1)$ demeaned asset returns

$$r_{i,t+1}, i = 0, ..., n$$

In the case of a one-factor model, the first return $r_{0,t+1}$ will play the role of the source return while we have n target returns $r_{i,t+1}, i = 1, ..., n$.

In the case of a two-factor model, the first return $r_{0,t+1}$ as well as the last return $r_{n,t+1}$ will play the role of the two source returns while we have $(n-1)$ target returns $r_{i,t+1}, i = 1, ..., n-1$. In other words, we live in this case with a dual notation for the two source returns:

$$\begin{aligned} r_{0,t+1} &= r_{01,t+1} \\ r_{n,t+1} &= r_{02,t+1} \end{aligned}$$

Generally speaking, $\{r_{i,t+1}, i \in I_n\}$ stands for the set of target returns with: $I_n = \{1, 2, ..., n\}$ for a one-factor model, and $I_n = \{1, 2, ..., n-1\}$ for a two-factor model.

3.1 Estimation of factor loadings

Following Doz and Renault (2006), standard and efficient GMM inference can be performed thanks to the complete set of conditional moment restrictions implied by our factor model:

$$E_t\left[r_{j,t+1}\left(r_{i,t+1}-\sum_{k=1}^{K}b_{i,k}r_{0k,t+1}\right)\right]=c_{i,j},\forall j=0,1,...,n,\forall i\in I_n$$

Recall that $I_n=\{1,2,...,n\}$ when $K=1$ and $I_n=\{1,2,...,n-1\}$ when $K=2$.

For a given vector z_t of instruments, we end up with the following set of unconditional moment restrictions for each asset $i\in I_n$:

$$E\left[z_t r_{j,t+1}\left(r_{i,t+1}-\sum_{k=1}^{K}b_{i,k}r_{0k,t+1}\right)\right]=c_{i,j}E(z_t),\forall j=0,1,...,n \tag{7}$$

In all our applications, our vector z_t of instruments is the most usual to capture conditional heteroskedasticity, namely the $(n+2)$-dimensional vector containing the constant and the squared current returns $r_{j,t}^2, i=0,1,...,n$. In other words, for each asset $i\in I_n$, (7) entails $(n+1)(n+2)$ unconditional moment restrictions.

For each target asset $i\in I_n$, the above moment restrictions can be written as a multivariate linear regression model as follows:

$$(r_{t+1}\otimes z_t)\,r_{i,t+1}=\sum_{k=1}^{K}b_{i,k}\,(r_{t+1}\otimes z_t)\,r_{0k,t+1}+[Id_{n+1}\otimes z_t]\,c_{i\bullet}+\varepsilon_{i,t+1} \tag{8}$$

where $r_{t+1}=(r_{j,t+1})_{0\le j\le n}$, $\varepsilon_{i,t+1}$ is a $(n+1)(n+2)$-dimensional martingale difference sequence, $c_{i\bullet}=(c_{i,j,})_{0\le j\le n}$ and Id_{n+1} stands for the identity matrix of dimension $(n+1)$.

Therefore, for each asset $i\in I_n$, efficient GMM on moment restrictions (7) is akin to solving some linear equations about sample averages over observations $t=1,2,...,T$ of equations (8). Of course, there is a non-zero correlation between equations for different assets, and thus some efficiency gains would be possible by performing joint GMM on the set of all assets together. Zellner's theorem does not apply since for each equation, OLS is not efficient. However, we will overlook the possible efficiency gain of grouping and we will estimate regression equations asset by asset.

It is worth noting that, in spite of this simplification, our approach remains fully multivariate since for each target asset, all asset returns are considered as potential instruments for capturing its conditional heteroskedascity. In particular, we want to emphasize that, as far as heteroskedasticity dynamics for each asset return is concerned, even our one-factor model is arguably less restrictive than a set of univariate GARCH models for each asset. More precisely, while a univariate GARCH(1,1) model would impose that:

$$E_t(r_{i,t+1}^2) = \omega_i + \gamma_i E_{t-1}(r_{i,t}^2)$$

(where the volatility persistence γ_i stands for the sum of the two GARCH parameters), we write instead:

$$E_t(r_{i,t+1}^2) = c_{i,i} + b_i E_t(r_{i,t+1} r_{0,t+1})$$

Then, for instance a linear forecasting model of the source return by the target return:

$$E_t[r_{0,t+1} \, | r_{i,t+1}] = g_{0,i,t} + d_{0,i}(r_t) r_{i,t+1}$$

would leave the door fully open for the specification of conditional heteroskedasticity:

$$E_t(r_{i,t+1}^2) = \frac{c_{i,i}}{1 - d_{0,i}(r_t)}$$

Of course, our model also imposes some restrictions on conditional covariances such as:

$$E_t(r_{i,t+1} r_{j,t+1}) = c_{i,j} + b_i E_t(r_{j,t+1} r_{0,t+1})$$

but this sounds much less restrictive than imposing say, constant conditional correlation between univariate GARCH(1,1), which can be represented by the particular case:

$$d_{0,j}(r_t) = \rho_j \left[\frac{E_t(r_{0,t+1}^2)}{E_t(r_{j,t+1}^2)} \right]^{1/2}$$

The bottom line is that the factor model, designed for the purpose of capturing contagion, is much less restrictive than it may first appear. It sets the focus on the expected commonalities between the risk components of different assets, but leaves the door open for any forecasting model of individual return volatility. Unsurprisingly at least when allowing for structural changes in the parameters, one-factor models (or possibly two-factor models) are not rejected by the data in our empirical examples in most circumstances.

3.2 Decomposition of variance

We identify the decomposition of variance between factor(s) and residual term by the choice of some $\alpha \in [\max_{1\leq j\leq K} \bar{\alpha}_j, 1]$ such that for each factor j and source return $r_{0j,t+1}$

$$\begin{aligned} Var(f_{j,t+1}) &= \alpha Var(r_{0j,t+1}) \\ Cov\,(r_{i,t+1}, r_{0j,t+1}) &= \alpha b_{i,j} Var(r_{0j,t+1}) + \omega_{i0}, \forall i \in I_n \end{aligned}$$

Consider for each target asset $i \in I_n$ an augmented set of moment restrictions as follows:

$$\begin{aligned} E\left[z_t r_{j,t+1}\left(r_{i,t+1} - \sum_{k=1}^{K} b_{i,k} r_{0k,t+1}\right)\right] &= c_{i,j}E(z_t), \forall j = 0, 1, ..., n \\ E\left[r_{0k,t+1}\left(r_{i,t+1} - \alpha b_{i,k} r_{0k,t+1}\right)\right] &= \omega_{i,0k}, 1 \leq k \leq K \end{aligned} \tag{9}$$

While (9) entails two more moment restrictions than (7), it does not modify the asymptotic variance of an efficient estimator of factor loadings $b_{i,k}, 1 \leq k \leq K$, (or the residual parameters $c_{i,j}$) because the additional moment restrictions just identify the additional parameters $\omega_{i,0k}, 1 \leq k \leq K$. However, the GMM estimates of the latter parameters will be more efficient than their naive sample counterparts obtained by plugging in the GMM estimates of $b_{i,k}$ deduced from (7) because (7) overidentifies the unknown parameters and thus provides "implied probabilities" to improve the estimates of additional moments ; see e.g. Back and Brown (1993).

Using the two factor model as an example, the augmented set of moment conditions (9) can be rewritten as follows:

$$\begin{aligned} &\begin{bmatrix} (r_{t+1} \otimes z_t)\, r_{i,t+1} \\ r_{01,t+1} r_{i,t+1} \\ r_{021,t+1} r_{i,t+1} \end{bmatrix} \\ &= \begin{bmatrix} (r_{t+1} \otimes z_t)\, r_{01,t+1} & (r_{t+1} \otimes z_t)\, r_{02,t+1} & [Id_{n+1} \otimes z_t] & 0 & 0 \\ \alpha r_{01,t+1}^2 & 0 & 0 & 1 & 0 \\ 0 & \alpha r_{02,t+1}^2 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} b_{i1} \\ b_{i2} \\ c_{i\bullet} \\ \omega_{i,01} \\ \omega_{i,02} \end{bmatrix} \\ &\quad + \begin{bmatrix} \varepsilon_{i,t+1} \\ \eta_{i1,t+1} \\ \eta_{i2,t+1} \end{bmatrix} \end{aligned} \tag{10}$$

Note that while $\varepsilon_{i,t+1}$ is a martingale difference sequence, $\eta_{i,t+1}$ will generally require correction for serial correlation. For sake of simplicity, we use a HAC estimator for the whole set of moment conditions, overlooking the fact that we know that a subvector is a martingale difference sequence. This is of course immaterial asymptotically and our actual sample sizes are large. If $\hat{\Sigma}_T^{(i)}$ stands for a HAC estimator computed from OLS residuals in (10), our efficient GMM estimator of $\theta^{(i)} = [b_{i1}, b_{i2}, c_{i0}, c_{i1}, ..., c_{in}, \omega_{i,01}, \omega_{i,02}]'$ will be:

$$\hat{\theta}_T^{(i)} = \left[\bar{X}_T'(\hat{\Sigma}_T^{(i)})^{-1}\bar{X}_T \right]^{-1} \bar{X}_T'(\hat{\Sigma}_T^{(i)})^{-1}\bar{Y}_T^{(i)}$$

with:

$$\bar{Y}_T^{(i)} = \frac{1}{T}\sum_{t=1}^{T} \begin{bmatrix} (r_{t+1} \otimes z_t)\, r_{i,t+1} \\ r_{01,t+1} r_{i,t+1} \\ r_{02,t+1} r_{i,t+1} \end{bmatrix} \quad (11)$$

$$\bar{X}_T = \frac{1}{T}\sum_{t=1}^{T} \begin{bmatrix} (r_{t+1} \otimes z_t)\, r_{01,t+1} & (r_{t+1} \otimes z_t)\, r_{02,t+1} & [Id_{n+1} \otimes z_t] & 0 & 0 \\ \alpha r_{01,t+1}^2 & 0 & 0 & 1 & 0 \\ 0 & \alpha r_{02,t+1}^2 & 0 & 0 & 1 \end{bmatrix} \quad (12)$$

We maintain the same identification assumption $A(\alpha)$ for the two periods (see empirical section for discussion of the way to pick a specific value of α). This assumption identifies in particular the residual covariance parameters $\omega_{i,01}, \omega_{i,02}, i = 1, ..., n-1$. Then, using moment conditions (9), we can estimate jointly by GMM for each period the asset i parameters $\theta_i = (b_i, c_{i\bullet}, \omega_{i0}), i = 1, ..., n-1$ denoted respectively as:

$$\theta_L^{(i)} = (b_{i,L}, c_{i\bullet,L}, \omega_{i0,L}), i = 1, ..., n-1$$
$$\theta_H^{(i)} = (b_{i,H}, c_{i\bullet,H}, \omega_{i0,H}), i = 1, ..., n-1$$

3.3 Identifying contagion

As already mentioned, contagion will be identified by comparing the model parameter values between a crisis period and a non-crisis period. We will always assume that the possible break point date is known, such that we have a non-crisis period for observations at dates $t = 1, ..., T_L$, and a crisis period for observations at dates $t = (T_L + 1), ..., (T_L + T_H)$ .We assume that each sample size goes to infinity such that valid asymptotic theory can be used for inference within each sample.

3.3.1 Estimating the parameters

We have three kinds of parameters to estimate:

(i) Model free parameters:

We estimate within each sample the correlation (including the Rigobon's corrected correlation (4)) and regression coefficients between each asset return and the source by the sample counterparts. We end up with consistent asymptotically normal estimators for each target asset $i \in I_n$ an each source asset j ($1 \leq j \leq K$) of:

$$(\rho_{ijL}, \rho_{ijH}, \tilde{\rho}_{ijH})$$
$$(\beta_{ijL}, \beta_{ijH})$$

(ii) Model-based parameters:

We maintain the assumption that the one or two factor model, or at least its characterization through the moment conditions (9), is well-specified, for each of the two periods. Using by GMM we can then estimate, for each period and each target asset $i \in I_n$ its parameters $(b_{ij}, c_{ij}), 1 \leq j \leq K$ denoted respectively as:

$$(b_{ij,L}, c_{ij,L})$$
$$(b_{ij,H}, c_{ij,H})$$

Note that we use the asymptotic theory of GMM for inference about these parameters in the context where the length T of the time series is seen as going to infinity, while the number, n, of assets is considered as given. Typically, this implies in practice, that we have to work with n small with respect to T. In our examples n is always smaller than 10, while T is always several hundred.

(iii) Decomposition of variance parameters:

We maintain the same identification assumption $A(\alpha)$ for the two periods, and this assumption identifies in particular the residual covariance parameters $\omega_{ij0}, i = 1, ..., n$. Then, using moment conditions (9), we can estimate jointly by GMM for each period the target asset i parameters $\theta_{ij} = (b_{ij}, c_{ij}, \omega_{ij0}), 1 \leq j \leq K$, denoted respectively as:

$$\begin{aligned} \theta_L^{(ij)} &= (b_{ij,L}, c_{ij,L,}\omega_{ij0,L}), i \in I_n, 1 \leq j \leq K \\ \theta_H^{(ij)} &= (b_{ij,H}, c_{ij,H,}\omega_{ij0,H}), i \in I_n, 1 \leq j \leq K \end{aligned}$$

It is worth recalling that the parameters $(b_{ij}, c_{ij\bullet}), i, j = 1, ..., n, i \neq j$ are actually estimated with the same asymptotic efficiency whether they are identified by just the moment conditions (7) or by the augmented set of moment conditions (9) that incorporates the decomposition of variance. For the sake of convenience, we set the focus on (9) for statistical inference. Once all parameters are estimated for the two periods, it is easy to run some Wald tests to test for structural change between non-crisis and crisis periods. However, as far as model-based parameters are concerned, GMM inference paves the way for several structural stability tests reviewed in the next section (see Hall (2005) section 5.4 for a comprehensive survey).

3.3.2 Testing hypotheses about structural stability

Since our focus of interest is the characterization of the differences between crisis and non-crisis periods, we never try to estimate or to test for the specification of a model pretending that there is no such thing as a structural change between the two periods. Therefore, we first estimate separately a one or two factor model in each period and run the corresponding overidentification test. More precisely, we compute for each target asset $i \in I_n$ the Hansen's J-specification test statistics:

$$J_{i,L} = T_L \left(\bar{Y}^{(i)}_{T_L,L} - \bar{X}_{T_L,L} \hat{\theta}^{(i)}_{T_L,L} \right)' (\hat{\Sigma}^{(i)}_{T_L,L})^{-1} \left(\bar{Y}^{(i)}_{T_L,L} - \bar{X}_{T_L,L} \hat{\theta}^{(i)}_{T_L,L} \right) \quad (13)$$

for the factor model of the non-crisis period as well as the Hansen's J test statistics:

$$J_{i,H} = T_H \left(\bar{Y}^{(i)}_{T_H,H} - \bar{X}_{T_H,H} \hat{\theta}^{(i)}_{T_H,H} \right)' (\hat{\Sigma}^{(i)}_{T_H,H})^{-1} \left(\bar{Y}^{(i)}_{T_H,H} - \bar{X}_{T_H,H} \hat{\theta}^{(i)}_{T_H,H} \right) \quad (14)$$

for the factor model of the crisis period. Since the moment conditions for decomposition of variance just identify the residual variance parameters $\omega_{i,0k}$, we have actually to assess overidentification through the $(n+1)(n+2)$ unconditional moment restrictions (7) that are used to identify the K factor loadings $b_{i,k}, 1 \leq k \leq K$ as well as the $(n+1)$ parameters $c_{i,j}, j = 0, 1, ..., n$. Therefore, when estimating a one factor model, under the null hypothesis that this model is well-specified:

$$J_{i,L}, J_{i,H} \longrightarrow_d \chi^2 \left[n(n+2) \right]$$

while, with a two-factor model:

$$J_{i,L}, J_{i,H} \longrightarrow_d \chi^2 \left[n(n+2) - 1 \right]$$

Even if a one factor (or a two factor) model is valid in each single period, the value of the factor loadings may change. For this reason we apply the Ghysels and Hall's predictive test, which looks like an overidentification test in the crisis period but uses the GMM estimator of the non-crisis period. More precisely the test statistic for a given target asset $i \in I_n$ is:

$$GH_i(H/L) = T_H \left(\bar{Y}^{(i)}_{T_H,H} - \bar{X}_{T_H,H}\hat{\theta}^{(i)}_{T_L,L} \right)' (\hat{\Omega}^{(i)}(H/L))^{-1} \left(\bar{Y}^{(i)}_{T_H,H} - \bar{X}_{T_H,H}\hat{\theta}^{(i)}_{T_L,L} \right)$$

where:

$$\hat{\Omega}^{(i)}(H/L) = \hat{\Sigma}^{(i)}_{T_H,H} + \frac{T_H}{T_L} \bar{X}_{T_H,H} \left(\bar{X}_{T_L,L} (\hat{\Sigma}^{(i)}_{T_L,L})^{-1} \bar{X}'_{T_L,L} \right)^{-1} \bar{X}'_{T_H,H}$$

This test statistic examines the validity of the orthogonality conditions in the second period, given that the conditions hold in the first period at some specific value $\theta^{(i)}_L$ of the parameters. Let us denote by $H_{0,K}(H/L)$ this null hypothesis under the maintained hypothesis that a K-factor model is valid in the non-crisis period. Then, Theorem 2 of Ghysels and Hall (1990) tells us that under the null hypothesis $H_{0,K}(H/L)$:

$$GH_i(H/L) \longrightarrow_d \chi^2 \left[(n+1)(n+2) + K \right]$$

Note that the number of degrees of freedom is larger than for the overidentification J-test because we do not subtract the number of estimated parameters; rather we adjust the weighting matrix to account for the variance of $\bar{X}_{T_H,H}\hat{\theta}^{(i)}_{T_L,L}$. We also account for the fact that our unconditional moment restrictions entail not only the $(n+1)(n+2)$ equations to identify factor loadings but also the K equations to identify $\omega_{i,0k}, 1 \leq k \leq K$. Note that Ghysels and Hall (1990) prove this result by assuming that T_L goes to infinity and $T_H = \lambda T_L$ for some $\lambda > 0$. The result is clearly valid more generally when both T_L and T_H go to infinity and the ratio (T_H/T_L) has a finite limit.

The power of the Ghysels and Hall's predictive test is fully based on the ability of the entire parameter vector in the non-crisis period

$$\theta^{(ij)}_L = (b_{ij,L}, c_{ij,L}, \omega_{ij0,L}), 1 \leq j \leq K$$

to reproduce the idiosyncratic volatility of the target asset i and its comovements with other assets during the crisis period. For the purpose of economic policy implications, it may be even more important to set the focus on the stability of the factor loadings.

Hall (2005) states the convenient result that under the null hypothesis of no change, the two J-test statistics on the one hand and the Wald test statistics comparing parameter values on the other hand are asymptotically independent. We will set the focus on the Wald test of no change in the factor loadings. In other words, for each target asset $i \in I_n = 1, 2, ...n-1$, the Wald test statistic will be:

$$\xi_{ij}^{W} = (T_L + T_H)\left(\hat{b}_{i,H} - \hat{b}_{i,L}\right)'\left[\hat{V}_{i,b}(H/L)\right]^{-1}\left(\hat{b}_{i,H} - \hat{b}_{i,L}\right)$$

where:

$$\hat{V}_{i,b}(H/L) = \left[\frac{T_L}{T_L + T_H}\right]^{-1}\hat{\Omega}(\hat{b}_{i,L}) + \left[\frac{T_H}{T_L + T_H}\right]^{-1}\hat{\Omega}(\hat{b}_{i,H})$$

where $\hat{\Omega}(\hat{b}_{i,L})$ (resp. $\hat{\Omega}(\hat{b}_{i,H})$) stands for the estimated asymptotic variance of the GMM estimator$\sqrt{T_L}\hat{b}_{i,L}$ (resp. $\sqrt{T_H}\hat{b}_{i,H}$). As usual, the test statistic will be asymptotically $\chi^2(1)$ (resp. $\chi^2(2)$) under the null hypothesis that the factor loading of the one factor (resp. two factor) model had not changed between the non-crisis and the crisis period.

4 Empirical Implementation

A challenge to testing for contagion is the variety of assets, markets and time periods in which these problems apply. Here we provide three different examples to illustrate the outcomes for evidence of contagion when the tests are applied to the change to the underlying factor loadings rather than the correlation coefficients. The illustrations cover different markets, assets and sample periods. We consider currency markets during the Asian financial crisis of 1997-1998, US equity industry sectors for 2007-2009 and the CDS market for European sovereigns over the period of 2008-2013. These examples demonstrate the behavior of a single factor model, the role of the reflection problem and the evidence for a two factor model respectively. In each case our selection of precise demarcation between the non-crisis and crisis period draws on the commonly accepted dates in the existing literature.[3]

The descriptive statistics of daily returns in each of the currency and equity markets, and the daily changes in 5-year CDS spreads, for the non-crisis and crisis periods of each example are given in Table 1. Data sources are provided in Appendix 1. Each example shows the typical increase in unconditional variance of the asset returns (or changes in CDS spreads) and the associated increase in range of returns in the form of both more extreme maxima and minima during the crisis period than the non-crisis period.

Having nominated our mimicking factors, the values of α_i are calibrated from univariate GARCH(1,1) estimates of the minimum conditional variance for the source asset over the non-crisis period.[4] Correction for serial correlation is not necessary for the majority of the model as it relies on conditional moments. However, it may influence the unconditional moments. The results in this paper focus on 5 day moving averages, representing a one-week horizon, but are insensitive to other choices for the parameter estimates.[5]

In deciding how many factors will be included in our model we include both economic and statistical evidence. Statistically, we want the model specification to reduce the evidence of GARCH in the crisis period once the common factor effect has been removed. Thus we test for improvements using the ARCH-LM test on the residuals of the model in the crisis period over the raw data. However, this test only provides an indication, at least in part because it has no mechanism for balancing the multiple results from multiple target assets. Thus we follow the extant literature on testing for contagion and nominate a potential source asset using the history of the crisis events. Having chosen the mimicking factors we then subject the framework to the specification tests for structural stability.

The estimated results for the first two applications provide evidence that the single factor framework passes the overidentification tests in both the non-crisis and crisis periods of the sample, but in the third example a two factor framework is

[3]Exogenous dating choices for crisis periods are common in the literature, and usually relate to observed events. For a few recent attempts to endogenously choose both crisis dates and explore contagion effects see Dungey et al (2015) and Contessi et al (2014).

[4]However, the empirical results are not sensitive to alternative choices which conform to the restrictions discussed in Section 2.4.

[5]Results for alternative smoothing parameters are available from the authors or by altering the smoothing parameter in the accompanying code appropriately.

empirically and economically preferable. The Ghysels-Hall predictive tests suggest that in the vast majority of cases the estimated parameters from the non-crisis period do not provide a good representation of the second period loadings, implying that something has changed over the sample. Tests of structural changes reject the hypothesis that the underlying parameters remain the same.

The necessity of using our model based approach to examine changes in b_{ij} as opposed to β_{ij} is evident across all the examples. In each application the estimated values of $\omega_{ij,0}$ in the non-crisis period are statistically significant for every asset, and their absolute values increase in the crisis periods (although sometimes with a change in sign). Typically the changes in b_{ij} are not identified by changes in β_{ij}, due to the role of $\omega_{ij,0}$, pointing to the importance of the decomposition (6) in correctly detecting contagion effects.[6]

4.1 Contagion between Currencies: 1997-1998 East Asian crisis

The first example concerns the behavior of currency markets during the East Asian crisis of 1997-1998. This crisis is commonly dated from the float of the Thai baht on July 2, 1997 and we nominate a non-crisis period from January 2,1995 to July 1, 1997 ($T_L = 650$) and crisis period from July 2, 1997 to August 31,1998 ($T_H = 305$). The data are drawn from Dungey and Martin (2007) and consist of daily US dollar exchange rate returns for the Thai baht, Indonesian rupiah, Malaysian ringgit and Australian dollar.

Our economic prior is that the Thai baht provides an appropriate source factor for this example. Table 2 reports the ARCH-LM test results for the model using each of the alternative assets as the potential source factor and supports the choice of the Thai baht as the most effective mimicking factor in eliminating the ARCH effects present in the data; reducing it to insignificance in Australia and Malaysia and reducing the evidence for Indonesia. It is worth emphasizing that our approach picks as a leading factor the currency of a small country instead of the natural

[6]In each example we also fitted the estimated factor loadings for the crisis period to the non-crisis data and conducted ARCH-LM tests on the resulting residuals. In each case the results were uniformly inferior in capturing the non-crisis period conditional volatility structure than the estimated non-crisis factor model. These figures are not reported in the paper but are included in the output of the accompanying code or available from the authors on request.

choice of the Australian dollar. The empirical results below confirm that this provides a powerful tool to identify contagion.

The changes in correlation coefficients between each currency return and the Thai baht reported in the top panel of Table 3 show the typical Forbes and Rigobon (2002) result where unadjusted correlation coefficients increase statistically significantly between the non-crisis and crisis period, but this difference is insignificant after the heteroskedasticity adjustment, supporting the conclusion of no contagion. All unconditional beta coefficients β_i reported in the second panel of Table 2 significantly increase, giving an indication that there was indeed a contagion phenomenon that has been hidden by over-correction of the correlation coefficient, reflecting that a significant part of the increase in the volatility component is due to the idiosyncratic component (see Proposition 2.1).

Our modelling framework provides clear evidence of contagion from the Thai baht to the Indonesian rupiah, Malaysian ringgit and Australian dollar exchange rates in the form of a statistically significant increase in the factor loadings b_i between the non-crisis and crisis periods; see the last panel of Table 3, estimated with $\alpha = 0.3$. This evidence is completely consistent with the vast majority of the empirical studies on this crisis; see the summary in Dungey et al (2006). While the loadings on the Thai baht exchange rate as the mimicking factor change from significantly negative during the non-crisis period, $b_{i,L}$, to significantly positive in the crisis period, $b_{i,H}$, for Indonesia and Malaysia, they do not change sign in the Australian case. Instead, the Australian loading becomes absolutely smaller during the crisis period, but remains negative. This is a good illustration of the information content of the factor loadings b_i for identifying contagion.

During the crisis period, Indonesia and Malaysia were both affected as near-neighbors with potentially similar economic structural problems to Thailand, subjecting them to contagion channels through both regional proximity and the wake-up call of contagion, see Goldstein (1998). This is reflected in the change in the loadings; in the crisis period the loading $b_{i,H}$ for Indonesia is not only positive, but almost 30 times greater than the pre-crisis loading in absolute value. For Malaysia, it is correspondingly almost 10 times larger.

This striking evidence of contagion from Thailand to Indonesia and Malaysia is somewhat less compelling when looking at unconditional beta coefficients. While they do increase between non-crisis and crisis periods, it is by a smaller extent and is always positive. This is due to strongly positive regression coefficients between idiosyncratic components, even in the non-crisis periods, that are able to mask the actual behavior of factor loadings b_i. In other words, while these Asian currencies are all unconditionally positively correlated, the strength of their link with the common volatility factor has been magnified by the crisis.

4.2 Contagion between Industries in the US: 2007-2009 crisis

The second example is designed to highlight the reflection problem of Manski (1993) and considers contagion from the financial sector in the US to other sectors of the economy using daily returns in the S&P500 sector indices for the banking, insurance, industrials, health, utilities, food and information technology sectors from August 1, 2004 to June 30, 2009. We contrast results where the US banking sector or the insurance sector play the role of the source asset; contributing to the debate on the role of insurance in crises. The demarcation between non-crisis and crisis period is the date on which the European Central Bank first became active in extending its support to markets following the revelations of stress, splitting the sample at August 9, 2007, so that $T_L = 788$ and $T_H = 498$; see Bekaert et al (2014) and Contessi et al (2014). A detailed description of the events of the crisis may be found in sequential issues of the IMF Global Financial Stability Reports for 2008 and 2009. Although the consensus in the literature is that the source shock originated in banking, there is some dissent about the potential importance of insurance; see Chen et al (2014), Acharya and Richardson (2014), Dungey et al (2014) and Harrington (2009).

Contagion between industry sectors has not been extensively examined using the frameworks applied to either country-wide indices or other markets, although there is an earlier literature regarding the transmission of shocks related to bankruptcy announcements as in Lang and Stulz (1992). While the bankruptcy of other firms tends to spread negative effects to others via bank lending and economic linkages, more concentrated industries seem to have offsetting competitive advantages from the misfortunes of their rivals; Jorion and Zhang (2009), Hertzel and Officer (2012).

The ARCH-LM tests reported in Table 4 show that during the crisis period evidence of ARCH is reduced when the banking sector is the mimicking portfolio, but the evidence is not uniform across all sectors. Other possible mimicking factors shown in the Table also reduce ARCH in some assets and show increased ARCH in others. When insurance is the mimicking factor each of food, utilities and IT sectors all suffer an increase in evidence of ARCH; consequently the evidence is somewhat supportive of the use of the banking sector in favour of the insurance sector as the mimicking factor. For completeness we also present the parameter estimates for the case where food is the mimicking factor, as although there is little economic argument for the food sector as the source shock, it does result in the greatest reduction in ARCH in Table 4.

Table 5a reports the results for the case of banking as the mimicking portfolio. The first panel presents the typical Forbes and Rigobon (2002) result showing that although the unadjusted correlation coefficient rises in each sector, their adjusted correlation coefficients fall. The regression analysis parameters, $\beta_{i,L}$ and $\beta_{i,H}$, reported in the second panel of Table 5a are consistent with the separation of the financial sector from the real economy in the crisis period. Only in the insurance sector is $\beta_{i,H} > \beta_{i,L}$. Our model results report significant contagion from the banking sector as the source market to all other sectors of the economy, with $\alpha = 0.7$; see Table 5b. For each asset the non-crisis period loading on the banking factor, $b_{i,L}$, is positive and significant, consistent with the role that the banking sector plays in facilitating real economic activity by credit creation.

During the crisis period the loadings, $b_{i,H}$, change dramatically and become negative, although insignificant in each sector. The most dramatic changes in the loading point estimates occur for those industries most closely aligned with banking, and those where other instances of rescue packages were implemented; insurance and industrials.[7] However, in the crisis period the banking sector is no longer an individually significant factor in determining the volatility of the other assets. This dramatic change reflects contagion in the form of removed linkages between the banking sector and the other sectors; see also Gai and Kapadia (2010),

[7] Exemplars of rescued or assisted firms include the insurance giant AIG and TARP support to conglomerates such as GE, and 'cash for clunkers' style programs.

Bekaert et al (2014) and Dungey et al (2014). This form of contagion explains the poor performance of the ARCH-LM test to elicit the right factor. Typically, with the food sector as source, we do not have this effect of removed linkages, and thus the food sector gives the spurious feeling that it better explains the common source of volatility. The $\omega_{i,0}$ parameters increase substantially between the two periods, in four sectors moving from significantly negative to insignificantly positive (and while the standard deviation of banking returns increases almost four-fold – see Table 1 – the $\omega_{i,0}$ increase by a multiple of between 12 and 500 times their non-crisis values such that $\gamma_{i,H} > \gamma_{i,L}$ in each case). The contribution of the unexplained variation leads to incorrect conclusions by comparing either $\beta_{i,H}$ with $\beta_{i,L}$ or correlation coefficients when true interest is located in changes in the loading $b_{i,L}$ to $b_{i,H.}$

To explore the reflection problem we explore the possibility that the insurance sector may provide a better mimicking factor. We re-estimate the model using insurance as the mimicking factor asset using $\alpha = 0.8$. The model results are reported in Table 6 where the factor loadings in the non-crisis period, $b_{i,L}$, are positive significant and relatively small compared with the estimates during the crisis period, $b_{i,H}$,which are larger and less statistically significant or insignificant. Furthermore, although the estimates of $\omega_{i,0,L}$ are statistically significant and positive, those of $\omega_{i,0,H}$ are positive but insignificant and have at least doubled (except in the case of IT where there has been change in sign to negative). In consequence the increase in the volatility of the source shock for insurance results in $\gamma_{i,H} < \gamma_{i,L}$ for a number of sectors. We also implemented a 2 factor representation for this sector with both banking and insurance as mimicking portfolios. The deterioration in the performance of the model was marked with increased evidence of ARCH after applying the factor model to the data. In contrast to common wisdom a one factor model may display a better fit than a two factor model. (We do not report the results but they are available from the authors on request.)

Further, following the lead from the ARCH-LM test results in Table 4 which suggests that food may provide a useful mimicking factor, the second panel of Table 6 reports the corresponding parameter estimates. In this case the $b_{i,L}$ are positive for all but the banking sector - indicating that the food industry and banking industry do not respond in a similar manner - and in the crisis period the $b_{i,H}$ increases dramatically for all sectors, but the effect is insignificant for both the banking and food sectors. The significant t-test between the non-crisis and crisis period reflects the move from a significant to insignificant relationship with the food sector by the financial sector. The estimates of $\omega_{i,0,H}$ all increase in absolute value over those of $\omega_{i,0,L}$, but with varied changes in sign, indicating the highly varied idiosyncratic responses to the crisis conditions when food is the mimicking factor. In short, the results in Table 6 showing insurance and food as the potential mimicking factors point to the importance of the reflection problem and the need to clearly specify the hypothesis of the problem being addressed, both economically and statistically. Darolles and Gourieroux (2015) make a related point with respect to identification.

4.3 Contagion in the CDS market: European sovereigns 2008-2013

The final example examines the daily changes in 5-year CDS spreads for the sovereign debt of Ireland, Italy, Portugal and Spain and Germany during the Greek debt crisis and subsequent difficulties in European sovereign debt markets. The 5-year spread has been investigated as a benchmark in understanding crisis conditions in Wang and Bhar (2014) and Merton et al (2013).[8] As the source factor for Greece we construct a proxy measure of the risk premium it faces as the differential between the Greek and US 10 year bonds – daily CDS data for Greece are not available on the Markit database for the sample period.[9] The European sovereign debt markets remained relatively calm for the period September 9, 2008 to March 31, 2010 compared with the turmoil in equity and money markets. Our sample ends November 21, 2013 so that $T_L = 403$ and $T_H = 951$.

Figure 1 shows the constructed Greek-US spread and the other European CDS spreads over the sample period, where the vertical line represents the change between non-crisis and crisis sub-samples. The major increase in Greek spreads from the first quarter of 2010 onwards is clearly apparent, as are the less substantive rises for the other GIIPS countries. German CDS spreads rose only marginally in comparison - from an average of 35 basis points to 53 basis points between the

[8] CDS spreads at firm level are used to study contagion in Jorion and Zhang (2009).

[9] Arghyrou and Kontonikas (2012) demonstrate the strong relationship between Greek bond and CDS spreads using monthly data during this period.

non-crisis and crisis sample (see Table 1). [10]

We hypothesize that this scenario is represented with a two factor specification, with Greece and Germany forming the source assets. There are firm economic grounds to consider that our sample contains elements of reaction to both the unfolding uncertainty around Greece and the relatively safety of Germany. The political leadership of German Chancellor Angela Merkel in the crisis, the anchor that Germany provides to the Euro system, and its role in providing safe-haven assets, are motivating economic factors; see Arghyrou and Kontonikas (2012). Figure 1 supports the quite different evolution of the German CDS spreads – the descriptive statistics provided in Table 1 show that the variance of the changes in German CDS spreads barely changes between the non-crisis and crisis periods. Table 7 shows that while each of the possible assets used as in a single factor mimicking portfolio improves the ARCH-LM test outcomes, it does not make it insignificant, whereas when a two factor model with Germany and Greece as mimicking portfolios the conditional heteroskedasticity is insignificant. We implement the model with $\alpha = 0.8$, representing the maximum of the potential values for α estimated using univariate GARCH for the two source factors; the value using Germany of 0.8 exceeds that for Greece of 0.6 used in the one factor model.[11]

Table 8 shows that unlike the previous two examples in this case $b_{ij,L} > b_{ij,H}$. This recognizes that an increase in the CDS spread (as opposed to an increase in returns in the value of the domestic currency or the equity return) is associated with poor news for financial markets. During the non-crisis period the loadings on both factors are significant and positive for all markets and in the crisis period the loadings on both factors are significant and negative. The non-crisis period loadings on the Greek factor, $b_{GRE,i,L}$ display stronger loadings for the Portuguese and Spanish CDS than for the Irish and Italian, but all four of these loadings are

[10] The other clearly notable feature of the Greek spread data is the abrupt drop in the spread on March 12, 2012 associated with IMF approval of the Extended Fund Facility of 28 billion Euros on that date - this incident appears to have been largely idiosyncratic and related to fears of Greek exit from the Eurozone and is not represented in the behavior of the markets for sovereign debt of other European countries.

[11] We also implemented the two factor model with separate α_j for each of the source factors, with little change in the results. As long as the boundary condition is respected the results are insensitive to choice of α_j.Note also the difference here between our identification choice and that of Broto and Pérez-Quirós (2015) who rely on the emergence of a second factor during the crisis period to identify contagion.

positive and statistically significant. The loadings on the German factor in the non-crisis period, $b_{GER,i,L}$, are also positive and significant, largest for the Irish and Italian CDS and closer to unity for the Portuguese and Spanish. During the crisis period these loadings switch signs, and for Ireland, Italy and Spain are up to 50 percent higher than in the non-crisis period, reflecting their stronger connection to the German factor during this period (consistent with the reduced attention to Greece). This outcome is even more pronounced for Portugal. The Portuguese case is also shown to be separable from Greece in Caporin et al (2014). The decoupling of a previously grouped market in this example is reminiscent of the behavior of Mexican bonds which ceased following Latin American bond market factors after being upgraded to investment grade in 2000, and instead joined the North American group; see Rigobon (2002).

The interpretation of these results is aided by considering the relative changes in volatility in the returns between the non-crisis and crisis periods. For each of Ireland, Italy, Portugal and Spain the rise in volatility between the non-crisis period and crisis period was both less than that experienced in Greek spreads, and greater than that experienced in German spreads. Interpreting the changes in b_i in that light shows that the weights on the Greek factor, $b_{GRE,i}$, fall in absolute value because the markets disconnect from Greece. Greece's problems are seen as highly idiosyncratic. However, they do not escape completely as evidenced by the fact that there is a significant change in $b_{GRE,i}$, triggered by the Greek shock and thus consistent with contagion. In the case of the loadings on the German factor, $b_{GER,i}$, the increase in volatility for the other markets distances them somewhat from Germany, which as a safe-haven does not suffer from the same market reassessment of potential default risk. Thus, Greece and Germany represent extremes – Greece becomes a relatively more risky proposition and Germany a safe haven – and consequently the factor loadings on both assets become negative, representing the different ways in which they have separated from the remaining CDS markets in the sample.

5 Conclusion

This paper has proposed a new method to identify contagion. We detect the change in the loadings on the underlying source factor driving the financial measure of interest (be it returns or changes in spreads), obtaining identification by taking advantage of heteroskedasticity via the conditional volatility which characterizes most financial data at other than very low frequency. In this way we extend the insights of Bekaert et al (2014) who use conditional means on low frequency data to identify changes in factor loadings in detecting contagion. But we also take advantage of financial econometric developments in a GARCH common features framework and associated statistical inference techniques. In this way we are able to both detect contagion in higher frequency data, more akin to the typically 'fast and furious' nature of financial crises (Kaminsky and Reinhart, 2003), and to directly test for significant changes in the loadings on the chosen source asset. Our framework provides further information on whether we have made the correct choice of source for the mimicking factor to represent potential contagion effects in the crisis period in the form of ARCH-LM tests; when the model is well-specified ARCH effects should in general be reduced by the factor model during the crisis period. However, the reflection problem is inherent in this approach and encourages the use of both economic and statistical evidence. The model has an important advantage in dimensionality. Most contagion tests are difficult to estimate for large selections of assets. This framework can handle a substantial number of assets, and what is more, multiple sources of contagion simultaneously. It is worth emphasizing that our approach to identify factors is parsimonious in nature. A multifactor model is not necessarily preferred because it may introduce spurious heteroskedasticity when a single factor is sufficient to capture volatility dynamics in the underlying data. Future extensions are intended to consider the interactions of multiple asset classes, endogenous crisis dating, asynchronous timing in low and high volatility regimes in different assets and to address the issue of whether pre-crisis conditions are ever re-established following crisis events.

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6 Appendix 1: Data Sources

- **Asian currencies example:** Data are daily exchange rates against the US dollar for the sample period July 2, 1995 to August 31, 1998. The original data are sourced from Thomson Financial Datastream with codes as shown in the Table below. These data were also used in Dungey and Martin (2007).
- **US equity sectors example:** Data are daily equity indices for sectors of the US economy from the S&P500 for sample period August 2, 2004 to June 30, 2009. The original data are sourced from Thomson Financial Datastream with codes as shown in the Table below.
- **European CDS example:** Data are sourced from Markit for European sovereign USD issued CDS for the period September 15, 2008 to November 21, 2013. These data are available for purchase direct from the vendor. The bond yields used in the application are sourced from Thomson Financial Datastream with codes as given in the Table.

Asian currencies		US equities	
currency	Datastream code	sector	Datastream code
Australia	AUSTRUS	banking	SP5SIBB
Indonesia	USINDON	food	SP5SFRT
Malaysia	MYUSDSP	health	SP5EHCR
Thailand	USTHAIB	industry	SP5SEIND
		insurance	SP5GINS
		IT	SP5EINT
		utilities	SP5GUTL

European CDS	
Sovereign	Markit code
Ireland	IRELND_Rep Irlnd_4A88DE
Italy	ITALY_Rep Italy_4AB951
Germany	DBR_Fed Rep Germany_3AB549
Portugal	PORTUG_Rep Portugal_7AA999
Spain	SPAIN_Kdom Spain_8CA965
10 year bonds	Datastream code
Greek yield	GRBRYLD
US yield	USBRYLD

Table 1: Descriptive statistics for returns in non-crisis and crisis periods

	Example 1: Daily currency returns				*Example 2: Daily US equity sector returns*						
	AUD	IND	MYR	THB	Banking	Food	Health	Industry	I.T.	Insur	Utilities
non-crisis:		Jan 2, 1995 - July 1, 1997					Aug 2, 2004 - Aug 8, 2007				
mean	0.0045	0.0155	-0.0019	-0.0040	0.0724	0.0315	0.02387	0.0425	0.0362	0.0242	0.0655
max	2.5004	1.3649	0.7049	6.1875	4.9202	5.3536	2.9703	2.3912	3.1952	5.1788	3.6599
min	-1.6616	-0.7800	-1.1559	-4.5425	-5.6792	-5.3675	-2.7464	-2.9858	-4.0188	-7.1311	-3.9865
s.d.	0.4449	0.1562	0.1810	0.4914	1.2376	1.1089	0.6983	0.7402	0.9150	0.8678	0.8644
crisis:		July 2, 1997 - Aug 31, 1998					Aug 9, 2007 - June 30, 2009				
mean	0.0903	0.5024	0.1670	0.1772	-0.1784	-0.0932	-0.0564	-0.1300	-0.0653	-0.2180	-0.0743
max	3.045	31.5853	7.1165	17.0666	23.7262	7.8388	11.7131	9.5164	11.4610	16.3780	12.6840
min	-2.7257	-23.6063	-6.7593	-6.1702	-21.2461	-9.7258	-7.4152	-9.2150	-9.6701	-14.2109	-8.5299
s.d.	0.7590	5.8578	1.6658	1.9443	4.8275	2.1385	1.6838	2.3146	2.2612	3.8225	1.9640

	Example 3: Daily changes in sovereign CDS spreads					
	Ireland	Italy	Germ	Greece	Portugal	Spain
		Sept 15, 2008 - Mar 31, 2010				
mean	0.0027	0.0018	0.0006	0.0026	0.0025	0.0019
max	0.6018	0.2099	0.1110	0.4459	0.3350	0.1921
min	-0.2740	-0.1549	-0.1013	-0.4878	-0.2876	-0.1734
s.d.	0.0802	0.0475	0.0199	0.1124	0.0554	0.0476
		Apr 1, 2010 - Nov, 21, 2013				
mean	-0.0001	0.0008	-0.0001	0.0033	0.0023	0.0005
max	1.1379	0.7180	0.1175	6.9930	1.77066	0.5921
min	-1.5245	-0.7404	-0.1336	-27.4580	-1.6724	-0.6960
s.d.	0.1670	0.1277	0.0216	1.0223	0.2526	0.1279

Table 2: ARCH-LM tests for the Asian currencies crisis period using raw data and model residuals with different mimicking factors.
Crisis period July 2, 1997 - December 31, 1998.

		Thailand	Indonesia	Malaysia	Australia
ARCH in the raw data in crisis					
	$\chi^2(1)$	1.8964	10.5641	1.9581	7.2559
	p-value	(0.1685)	(0.0012)	(0.1617)	(0.0071)
ARCH in residuals after model using mimicking factor of:					
Thailand	$\chi^2(1)$		5.1050	0.5244	1.2496
	p-value		(0.0239)	(0.4690)	(0.2636)
Indonesia	$\chi^2(1)$	1.0878		2.8205	8.0135
	p-value	(0.2970)		(0.0931)	(0.0046)
Malaysia	$\chi^2(1)$	1.5825	13.0699		2.0852
	p-value	(0.2084)	(0.0003)		(0.1487)
Australia	$\chi^2(1)$	0.9136	10.3961	1.3432	
	p-value	(0.3392)	(0.0012)	(0.2465)	

Table 3: Asian currencies against the US dollar: contagion from Thai baht. Non-crisis period January 2, 1995 - July 1, 1997. Crisis period July 2, 1997 - December 31, 1998.

		Indonesia	Malaysia	Australia
Correlation Coefficients				
non-crisis period	ρ_L	0.0297	0.0973	-0.0186
crisis period: not adjusted	ρ_H	0.3194	0.4903	0.2994
crisis period: FR adjusted	$\widetilde{\rho}_H$	0.0847	0.1406	0.0789
t-test of change	$\rho_H - \rho_L$	4.3219	6.2977	4.6985
	p-value	(0.0114)	(0.0040)	(0.0091)
t-test of change	$\widetilde{\rho}_H - \rho_L$	0.7920	0.6304	1.4019
	p-value	(0.2431)	(0.2866)	(0.1278)
Regression coefficients				
non-crisis period	$\beta_{i,L}$	0.0452	0.0478	-0.0105
	s.e.	(0.0174)	(0.0149)	(0.0354)
crisis period	$\beta_{i,H}$	0.9685	0.4219	0.1182
	s.e.	(0.1635)	(0.0428)	(0.0214)
J-test for one factor model				
non-crisis period	$\chi^2(15)$	0.8201	2.0646	0.1888
	p-value	(1.0000)	(1.0000)	(1.0000)
crisis period	$\chi^2(15)$	5.9532	4.5516	4.1963
	p-value	(0.9805)	(0.9953)	(0.9970)
Tests for structural stability				
Ghysels-Hall test	$\chi^2(21)$	781.1276	1286.4467	957.2358
	p-value	(0.0000)	(0.0000)	(0.0000)
Break in factor loadings	$\chi^2(1)$	1050.966	1534.1026	897.1300
	p-value	(0.0000)	(0.0000)	(0.0000)
parameter estimates				
non-crisis period	$b_{i,L}$	-0.0508	-0.0468	-0.3665
	s.e.	(0.0001)	(0.0000)	(0.0001)
	$\omega_{i,0,L}$	0.0101	0.0046	0.0030
	s.e.	(0.0000)	(0.0000)	(0.0000)
crisis period	$b_{i,H}$	1.4889	0.4659	-0.1172
	s.e.	(11.4371)	(1.0627)	(0.1072)
	$\omega_{i,0,H}$	0.2015	0.2452	0.0494
	s.e.	(3.0605)	(0.1885)	(0.0264)
t-test of change	$b_{i,H} - b_{i,L}$	2.3472	8.4118	40.5466
	p-value	(0.0503)	(0.0018)	(0.0000)

Table 4: ARCH-LM tests for US industrial sectors for the crisis period using raw data and model residuals with different mimicking factors.
Crisis period August 9, 2007 - June 30, 2009.

		Banking	Insurance	Industrials	Health	Utilities	Food	Info Tech
ARCH in the raw data in crisis								
	$\chi^2(1)$	6.1260	31.2394	4.0340	22.3827	18.4958	5.0645	4.7180
	p-value	(0.0133)	(0.0000)	(0.0446)	(0.0000)	(0.0000)	(0.0244)	(0.0298)
ARCH in residuals after model using mimicking factor of:								
Banking	$\chi^2(1)$		11.3038	5.0102	13.8225	10.9546	10.2199	3.7938
	p-value		(0.0008)	(0.0252)	(0.0002)	(0.0009)	(0.0014)	(0.0514)
Insurance	$\chi^2(1)$	5.1480		37.9830	11.8064	29.0175	8.8670	46.6494
	p-value	(0.0233)		(0.0000)	(0.0006)	(0.0000)	(0.0029)	(0.0000)
Industrials	$\chi^2(1)$	10.1200	3.6582		1.3027	0.6927	10.3462	4.4106
	p-value	(0.0015)	(0.0558)		(0.2537)	(0.4052)	(0.0013)	(0.0357)
Health	$\chi^2(1)$	3.3662	44.6622	34.1518		29.2348	4.5204	35.4428
	p-value	(0.0665)	(0.0000)	(0.0000)		(0.0000)	(0.0335)	(0.0000)
Utilities	$\chi^2(1)$	5.6339	18.5202	54.7497	2.6298		4.5273	21.8354
	p-value	(0.0186)	(0.0000)	(0.0000)	(0.1049)		(0.0333)	(0.0000)
Food	$\chi^2(1)$	4.6144	2.5445	4.9309	2.1458	0.3277		2.2555
	p-value	(0.0317)	(0.1107)	(0.0264)	(0.1430)	(0.5670)		(0.1331)
Info Tech	$\chi^2(1)$	7.7054	29.7879	9.5449	8.6193	8.7193	11.4718	
	p-value	(0.0055)	(0.0000)	(0.0020)	(0.0033)	(0.0033)	(0.0007)	

Table 5a: Sectors of the US economy: contagion from banking.
Non-crisis period August 2, 2004 - August 8, 2007. Crisis period August 9, 2007 - June 30, 2009.

		Insurance	Industrials	Health	Utilities	Food	Info Tech
A: Correlation Coefficients							
non-crisis period	ρ_L	0.6448	0.7262	0.5866	0.5028	0.4107	0.6781
crisis period: not adjusted	ρ_H	0.8408	0.7682	0.6948	0.6009	0.5835	0.7633
crisis period: FR adjusted	$\widetilde{\rho}_H$	0.3699	0.2940	0.2404	0.1892	0.1811	0.2897
t-test of change	$\rho_H - \rho_L$	7.9510	1.6576	3.2105	2.4587	4.0198	3.1013
	p-value	(0.0021)	(0.0980)	(0.0245)	(0.0455)	(0.0138)	(0.0266)
t-test of change	$\widetilde{\rho}_H - \rho_L$	-3.5708	-10.7349	-7.4264	-6.2838	-4.4015	-9.1620
	p-value	(0.0036)	(0.0009)	(0.0025)	(0.0041)	(0.0109)	(0.0014)
B: Regression Coefficients							
non-crisis period	$\beta_{i,L}$	0.4552	0.4359	0.3309	0.3525	0.3687	0.5008
	s.e.	(0.0191)	(0.0146)	(0.0162)	(0.0215)	(0.0290)	(0.0193)
crisis period	$\beta_{i,H}$	0.6661	0.3687	0.2424	0.2448	0.2587	0.3576
	s.e.	(0.0193)	(0.0138)	(0.0113)	(0.0146)	(0.0162)	(0.0136)
C: J-test for one factor model							
non-crisis period	$\chi^2(48)$	13.4400	7.3939	7.1093	5.5387	8.0015	8.0967
	p-value	(1.0000)	(1.0000)	(1.0000)	(1.0000)	(1.0000)	(1.0000)
crisis period	$\chi^2(48)$	22.9327	22.6335	35.4208	32.9703	36.2919	26.4114
	p-value	(0.9992)	(0.9993)	(0.9111)	(0.9517)	(0.8924)	(0.9952)
D: Tests for structural stability							
Ghysels-Hall test	$\chi^2(57)$	6684.9363	5899.9617	3129.2909	4342.3432	4290.7312	5367.5544
	p-value	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Break in factor loadings	$\chi^2(1)$	3259.7417	4762.4342	2290.1074	2923.3895	4994.9196	3541.8982
	p-value	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)

Table 5b: Sectors of the US economy: contagion from banking.
Non-crisis period August 2, 2004 - August 8, 2007. Crisis period August 9, 2007 - June 30, 2009.

		Insurance	Industrials	Health	Utilities	Food	Info Tech
E: Parameter estimates							
non-crisis period	$b_{i,L}$	0.4973	0.5234	0.7652	0.5071	0.8990	0.9114
	s.e.	(0.0161)	(0.0077)	(0.0089)	(0.0098)	(0.0174)	(0.0085)
	$\omega_{i,0,L}$	0.0071	0.0077	-0.0484	-0.0064	-0.0637	-0.0396
	s.e.	(0.0026)	(0.0012)	(0.0014)	(0.0015)	(0.0027)	(0.0013)
crisis period	$b_{i,H}$	-1.6073	-1.0278	-0.2532	-0.7620	-0.6651	-0.4247
	s.e.	(2.5636)	(0.6714)	(0.76620)	(0.8667)	(0.4521)	(0.8813)
	$\omega_{i,0,H}$	3.5852	2.0587	0.6047	1.1243	1.3067	1.2258
	s.e.	(5.4702)	(1.5009)	(1.4579)	(1.7884)	(1.0139)	(1.7373)
t-test of change	$b_{i,H} - b_{i,L}$	-18.2275	-51.2952	-29.5098	-32.5115	-76.7864	-33.6611
	p-value	(0.0002)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)

Table 6: Sectors of the US economy: contagion with alternative mimicking factors.
Non-crisis period August 2, 2004 - August 8, 2007. Crisis period August 9, 2007 - June 30, 2009.

		Banking	Insurance	Industrials	Health	Utilities	Food	Info Tech
Parameter estimates with insurance as mimicking factor								
non-crisis period	$b_{i,L}$	0.5199	-	0.0939	0.3250	0.1609	0.4973	0.3784
	s.e.	(0.0243)	-	(0.0069)	(0.0068)	(0.0091)	(0.0184)	(0.0086)
	$\omega_{i,0,L}$	0.0403	-	0.0442	0.0156	0.0224	-0.0079	0.0159
	s.e.	(0.0024)	-	(0.0007)	(0.0007)	(0.0010)	(0.0192)	(0.0091)
crisis period	$b_{i,H}$	0.4234	-	0.9418	0.7194	0.90096	0.4905	1.0862
	s.e.	(1.6954)	-	(0.4350)	(0.4631)	(0.4838)	(0.4863)	(0.5864)
	$\omega_{i,0,H}$	1.3456	-	0.1910	0.8747	-0.0600	0.1184	-0.0760
	s.e.	(2.0724)	-	(0.5049)	(0.5337)	(0.5830)	(0.5581)	(0.6662)
t-test of change	$b_{i,H} - b_{i,L}$	-1.2638	-	43.2789	18.9059	34.3616	-0.3134	26.7946
	p-value	(0.1478)	-	(0.0000)	(0.0002)	(0.0000)	(0.3872)	(0.0000)
Parameter estimates with food as mimicking factor								
non-crisis period	$b_{i,L}$	-0.0556	1.0855	0.0779	0.1564	0.6368	-	0.4374
	s.e.	(0.0239)	(0.0237)	(0.0008)	(0.0055)	(0.0085)	-	(0.0088)
	$\omega_{i,0,L}$	0.0720	-0.0433	0.0455	0.0344	-0.0052	-	0.0084
	s.e.	(0.0028)	(0.0028)	(0.0009)	(0.0008)	(0.0010)	-	(0.0010)
crisis period	$b_{i,H}$	0.8507	4.1010	2.8766	2.5680	2.8817	-	2.4410
	s.e.	(2.4542)	(4.6291)	(1.1585)	(1.1930)	(1.4451)	-	(1.6001)
	$\omega_{i,0,H}$	0.4849	-0.3148	-0.2984	-0.3190	-0.3157	-	-0.2547
	s.e.	(0.7982)	(1.4392)	(0.3769)	(0.3785)	(0.4523)	-	(0.4843)
t-test of change	$b_{i,H} - b_{i,L}$	8.1993	14.4639	53.6391	44.8819	34.4913	-	27.8021
	p-value	(0.0019)	(0.0004)	(0.0000)	(0.0000)	(0.0000)	-	(0.0000)

Table 7: ARCH-LM tests for CDS for the European debt crisis period using raw data and model residuals with different mimicking factors. Crisis period April 1, 2010 - November 21, 2013.

		Germany	Ireland	Italy	Greece	Portugal	Spain
ARCH in the raw data in crisis							
	$\chi^2(1)$	62.8478	42.0411	24.9851	0.3950	7.4800	35.1931
	p-value	(0.0000)	(0.0000)	(0.0173)	(0.5297)	(0.0062)	(0.0000)
ARCH in residuals after model using mimicking factor of:							
Germany	$\chi^2(1)$		36.7133	35.9137	7.6560	5.5689	25.8559
	p-value		(0.0000)	(0.0000)	(0.0057)	(0.0183)	(0.0000)
Ireland	$\chi^2(1)$	59.7140		15.3613	35.9649	4.3449	18.0686
	p-value	(0.0000)		(0.0001)	(0.0000)	(0.0371)	(0.0000)
Italy	$\chi^2(1)$	59.7140	35.9649		15.3613	4.3449	18.0686
	p-value	(0.0000)	(0.0000)		(0.0001)	(0.0371)	(0.0000)
Greece	$\chi^2(1)$	22.2626	36.4608	29.2930		19.3448	31.7535
	p-value	(0.0000)	(0.0000)	(0.0000)		(0.0000)	(0.0000)
Portugal	$\chi^2(1)$	59.7140	35.9649	15.3613	4.3449		18.0686
	p-value	(0.0000)	(0.0000)	(0.0001)	(0.0371)		(0.0000)
Spain	$\chi^2(1)$	59.7140	35.9649	15.3613	4.3449	18.0686	
	p-value	(0.0000)	(0.0000)	(0.0001)	(0.0371)	(0.0000)	
ARCH in residuals after model using 2 mimicking factors of Germany and Greece							
Germany&	$\chi^2(1)$		0.4089	0.4199		0.4171	0.4110
Greece	p-value		(0.5225)	(0.5170)		(0.5184)	(0.5214)

Table 8: Two factor model The European sovereign CDS: sources from Greece and Germany.
Non-crisis period September 15, 2008 - March 31, 2010. Crisis period April 1, 2010 - November 21, 2013.

		Ireland	Italy	Portugal	Spain
J-test for two factor model					
non-crisis period	$\chi^2(34)$	0.2058	0.1877	0.1914	0.1640
	p-value	(1.0000)	(1.0000)	(1.0000)	(1.0000)
crisis period	$\chi^2(34)$	0.0468	0.0624	0.0537	0.0600
	p-value	(1.0000)	(1.0000)	(1.0000)	(1.0000)
Tests for structural stability					
Ghysels-Hall test	$\chi^2(44)$	1234.9338	1418.6571	1309.7833	1306.0346
	p-value	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Break in factor loadings	$\chi^2(2)$	71.0385	185.4015	133.6123	246.8636
	p-value	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Regression Coefficients					
non-crisis period	$\beta_{GRE,i,L}$	0.1017	0.0977	0.1532	0.1282
	s.e.	(0.0306)	(0.0160)	(0.0198)	(0.0155)
	$\beta_{GER,i,L}$	2.0325	1.4007	1.3316	1.3520
	s.e.	(0.1730)	(0.0904)	(0.1116)	(0.0878)
crisis period	$\beta_{GRE,i,H}$	0.0094	0.0060	0.0123	0.0059
	s.e.	(0.0045)	(0.0029)	(0.0069)	(0.0031)
	$\beta_{GER,i,H}$	4.0652	4.1053	5.8908	3.8589
	s.e.	(0.2139)	(0.1380)	(0.1460)	(0.1460)
Parameter estimates					
non-crisis period	$b_{GRE,i,L}$	0.8717	0.6376	1.1725	1.2094
	s.e.	(0.0000)	(0.0000)	(0.0000)	(0.0000)
	$b_{GER,i,L}$	5.0838	2.8542	1.1287	1.3801
	s.e.	(0.0000)	(0.0000)	(0.0000)	(0.0000)
	$\omega_{GRE,i,0,L}$	-0.0009	-0.0003	-0.0011	-0.0013
	s.e.	(0.0000)	(0.0000)	(0.0000)	(0.0000)
	$\omega_{GER,i,0,L}$	-0.0001	-0.0000	0.0001	0.0009
	s.e.	(0.0000)	(0.0000)	(0.0000)	(0.0000)
crisis period	$b_{GRE,i,H}$	-0.0207	-0.0303	-0.0635	-0.0338
	s.e.	(0.0002)	(0.0002)	(0.0004)	(0.0001)
	$b_{GER,i,H}$	-7.9511	2.9525	-9.0794	-1.6482
	s.e.	(0.0351)	(0.0102)	(0.0479)	(0.0089)
	$\omega_{GRE,i,0,H}$	0.0076	0.0076	0.0119	0.0062
	s.e.	(0.0000)	(0.0000)	(0.0001)	(0.0000)
	$\omega_{GER,i,0,H}$	0.0007	-0.0000	0.0007	0.0002
		(0.0000)	(0.0000)	(0.0000)	(0.0000)

Figure 1: European Spreads:
5 year CDS premia for Ireland, Italy, Portugal,
Spain and Germany:
10 year bond spread for Greece over the US