Frictional Diversification Costs: Evidence from a Panel of Fund of Hedge Fund Holdings

Bernd Scherer[♦]

[♦] Joint paper Juha Juvänää. We would like to thank seminar participants at the 2013 Austrian Working Group on Banking and Finance for helpful comments. We are grateful Mikko Kauppila and Mikko Perttunen for helping us with the data processing. Juha Joenväärä, PhD, is an Assistant Professor at the University of Oulu and a Visiting Researcher at Imperial College, London. Juha.joenvaara@oulu.fi. Bernd Scherer (corresponding author), PhD, is Managing Director at Deutsche Asset Management and visiting Professor at WU, Vienna bernd-dr.scherer@db.com

Executive Summary

- Diversification is no free lunch. It requires size (assets under management) to offer diversification due to frictional diversification costs (due diligence costs)

- We find a positive log linear relation between the number of funds, n, and the assets under management for (hedge) fund of funds, aum:

$$n^2 \propto aum.$$

- The presented evidence is econometrically robust and consistent with a model of naive diversification under frictional diversification costs.

Motivation I: Identifying Over- or Under-diversification

- Previous work on FoFs is devoted mainly to one simple question: How many hedge funds are needed for a *diversified* fund of funds? Henker and Martin (1998), Amin and Kat (2002), Lhabitant and Learned (2002), and Brown et al. (2013) all use a simple two-step procedure to test for over- or underdiversification:

Step 1: Simulate random portfolios of increasing size (i.e., increasing number of equally weighted assets) and plot the evolution of volatility as a "diversification curve": a functional relationship between portfolio standard deviation and portfolio size.[1]

Step 2: Decide when the marginal improvement in the statistic derived during step 1 becomes "small".

To make a statement on over- or under-diversification, we need a model of optimal diversification in the presence of frictional diversification costs

[1] In the case of volatility, Elton and Gruber (1977) show that a closed-form solution exists. Yet the simulation aspect is useful for illuminating how this procedure might extend to measures of risk and performance that are more complicated.

Methods based on diversification curves have three major shortcomings:

1. No attempt is made to specify the frictional costs of adding another fund to a portfolio. In the absence of such costs it is always optimal to naively diversify across all possible investments.

2. Diversification curves do not account for the actual assets under management – even though fixed costs can be spread more easily across a large pool of assets. A decision maker seeking the optimal number of assets in which to invest $10 million versus $100 million should certainly receive a different answer in each case.

3. The reduction in volatility that diversification is intended to provide is most valuable for investors with high risk aversion. It is clear that investors with low risk aversion will be less willing to pay the diversification costs of reducing risks (i.e. reducing volatility by adding more funds).

Motivation II: How big is n in 1/n?

- DeMiguel et al. (2009b) show that equal weighting (1/*n*) is preferable to mean variance optimization if the Sharpe ratio differences between assets are small (when adjusted for sample size). However, none of the (1/*n*)-based papers discusses frictional diversification costs and so the optimal number of assets (*n*) is imposed rather than derived.

- Our research differs from that of authors who look to explain poorly diversified portfolios (portfolios with too little number of names) in terms of the behavioral shortcomings of private versus professional decision makers. Statman (2004) coined the term "behavioral portfolio theory": the attempt to (psychologically) rationalize the observed under-diversification of individual investors.

- Hedge fund, operational risk and role as financial intermediaries. Brown, Goetzmann, Liang, and Schwarz (2008b, 2009, 2012) find that larger intuitions enjoy economies of scale, enabling direct investment into relatively better performing hedge funds.

A Parsimonious Model of FoF Diversification

- Following Goldsmith (1976) and Scherer (2013), we assume a decision maker that will trade off the fund's marginal benefits from naive diversification (formulated as marginal risk reduction multiplied by risk aversion) against their marginal costs to diversify.

(1) $$n^{*} = \arg\max_{n} \mu - \lambda\sigma^2\left(n\right) - n\frac{f}{aum}$$

- Here $\sigma^2\left(n\right)$ denotes the risk (variance) of an equally weighted portfolio of size n, λ the investor's coefficient of relative risk aversion, and $\frac{f}{\mathrm{AuM}}$ the additional costs (i.e., fixed costs f per additional fund as a fraction of assets under management, AuM).

- Think of f as the costs of a due diligence report. The costs of exercising due diligence are far from trivial: estimates range between \$50,000 and \$100,000 (US).

- We can now write down the first-order condition of our investor above as

(2) $$\frac{f}{\mathrm{AuM}} = -\lambda\frac{d\sigma^2\left(n\right)}{dn};$$

The expected variance for an equally weighted portfolio is well known to be

(3) $$\sigma^2(n) = \frac{\overline{\sigma}^2}{n} + \left(1 - \frac{1}{n}\right)\overline{\sigma}^2\overline{\rho}\,,$$

where $\overline{\sigma}$ and $\overline{\rho}$ are the average volatility and correlation in the universe of investable assets. We can find an explicit solution for the marginal change in risk,

(4) $$\frac{d\sigma^2(n)}{dn} = \frac{1}{n^2}\overline{\sigma}^2(\overline{\rho} - 1)\,.$$

Substituting (4) into (2) yields $\frac{f}{\text{AuM}} = -\lambda\frac{1}{n^2}\overline{\sigma}^2(\overline{\rho} - 1)$, which can be solved for the optimal n:

(5) $$n^* = \sqrt{\lambda\overline{\sigma}^2(1 - \overline{\rho})\left(\frac{f}{\text{AuM}}\right)^{-1}}\,.$$

The optimal number of assets increases with rising risk aversion (λ), rising average volatility ($\overline{\sigma}^2$), falling average correlation ($\overline{\rho}$), falling frictional costs (f), and rising *value* of assets under management (AuM).

- For our naive investor facing frictional diversification costs, the risk of an optimally diversified portfolio (i.e., one for which the marginal benefits from diversification just equal the costs of diversifying) is found by substituting (5) into (3):

(6) $$\sigma^2\left(n^*\right) = \overline{\sigma}^2\overline{\rho} + \sqrt{\frac{\overline{\sigma}^2(1-\overline{\rho})(f/\mathrm{AuM})}{\lambda}}.$$

- The first term on the right-hand side of (6) represents the average covariance risk in the available asset universe. This is the minimal achievable risk for $n \rightarrow \infty$ – that is, in the absence of frictional costs or for investors who are infinitely averse to risk. The second term reflects the higher risk that results when a diversification strategy accounts for its associated frictional costs, since those costs preclude investors from diversifying to the theoretical maximum.

- Taking logs on both sides of (5) results in a linear model,

(7) $$\log(n) = a + b \cdot \log(\mathrm{AuM}) + \varepsilon,$$

where $a = 0.5 \cdot \log\left(\lambda\overline{\sigma}^2(1-\overline{\rho})/f\right)$ and $b = 0.5$.

- In order to test (7) on a panel data set (instead of using a cross sectional snapshot), we propose running a fixed-effects model of the form (where i denotes a specific FoF and t a particular moment in time):

$$\log\left(n_{it}\right) = a_i + b \cdot \log\left(\mathrm{AuM}_{it}\right) + \varepsilon_{it}, \tag{8}$$

- This choice is motivated by the possibility of omitted variable bias in (7). Equation (7) is not likely to hold when investors are strongly convinced of their own forecasting ability. Greater investment skills manifest as variation in individual effects (a_i). Our fixed-effects, panel data model uses cross-sectional units as controls.

- The consequences of unobserved investment skill should cancel out provided the effect of skill is constant (i.e., a fixed effect). Another neat side effect of our panel data model is that it corrects for alternative investment universes, clientele effects (risk aversion) and frictional diversification costs.

Data

- We use a panel of registered fund of (hedge) fund holdings for the period 2004–2012. A FoF may opt to register with the US Securities and Exchange Commission (SEC) under the Investment Company Act of 1940, thereby gaining wider distribution channels.

- Registered FoFs are considered to be closed-end funds and so are usually not listed on exchanges. Exactly as do mutual funds, registered FoFs must publicly disclose mandated filings, including quarterly disclosures of portfolio holdings and semi-annual financial statements.

- Following Aiken et al. (2012, 2013), we gather the underlying hedge fund holdings of our sample FoFs from SEC forms N-Q, N-CSRS, and N-CRS. The data in those filings enable us to create a panel of quarterly hedge fund holdings, for each FoF, that contains the current value of each position. Hence we can calculate each FoF's total assets under management (AuM) and number of hedge funds (n) on a quarterly basis.

- It is important to emphasize that commercial hedge fund databases do not provide panel data about the number of underlying hedge funds in which a FoF has invested. Therefore, it would be impossible to test our model predictions of frictional diversification costs if we were limited to using data obtained from commercial databases.

Table 1

Data Characteristics

Funds of funds are sorted (by the number of available observations) into five sample size groups. For each group we report the number of FOFs it contains, the average number of observations, the average of $\log(\text{AuM})$, and the average number of holdings. We also estimate the mean group estimator for the average regression slope in $\log\left(n_i\right) = a_i + b_i \log\left(\text{AuM}_i\right) + e_i$ for all FoFs in each group. Given T_i observations (for FoF i), the mean group estimator for all FoFs in group k is given by $\hat{b}_k = \left(\sum_{i=1}^{N(k)} T_i \hat{b}_i\right)\left(\sum_{i=1}^{N(k)} T_i\right)^{-1}$ for $N\left(k\right)$ the number of FoFs in the group. The variance of the mean group estimator is given by $\text{var}\left(\hat{b}_k\right) = \left(\sum_{i=1}^{N(k)} T_i^2 \,\text{var}\left(\hat{b}_i\right)\right)\left(\sum_{i=1}^{N(k)} T_i^2\right)^{-1}$.

	$\#\text{Obs.} < 4$	$4 \leq \#\text{Obs.} < 8$	$8 \leq \#\text{Obs.} < 12$	$12 \leq \#\text{Obs.} < 20$	$20 < \#\text{Obs.}$
Number of FoFs, $N\left(k\right)$	49	27	17	20	14
Average number of observations	1.86	5.67	9.47	15.3	23.93
Average of $\log(\text{AuM})$	17.63	18.23	18.71	18.1	18.63
Average number of holdings, n	18.91	25.72	26.61	21.2	31.14
Mean group estimator (MGE) of $\hat{b}$	0.12	0.49	0.66	0.55	0.37
Standard Deviation of MGE, $\sigma\left(\hat{b}\right)$	0.21	0.27	0.09	0.03	0.01

Figure 1

Number of Holdings per FoF

The boxplots in this figure represent the distribution of the number of hedge fund holdings for each of our 127 FoFs. Entries are sorted from left to right by the number of available fund observations. Printed at the top of each grouping are the number of quarterly observations available for – and the number of FoFs belonging to – that group.

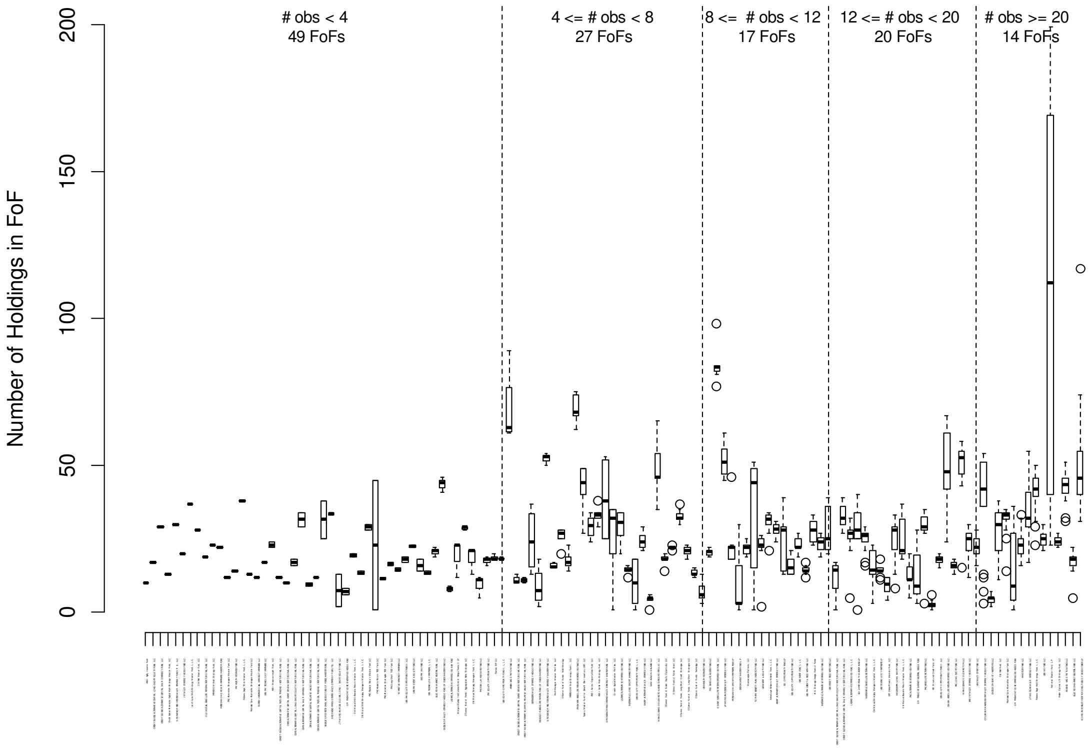
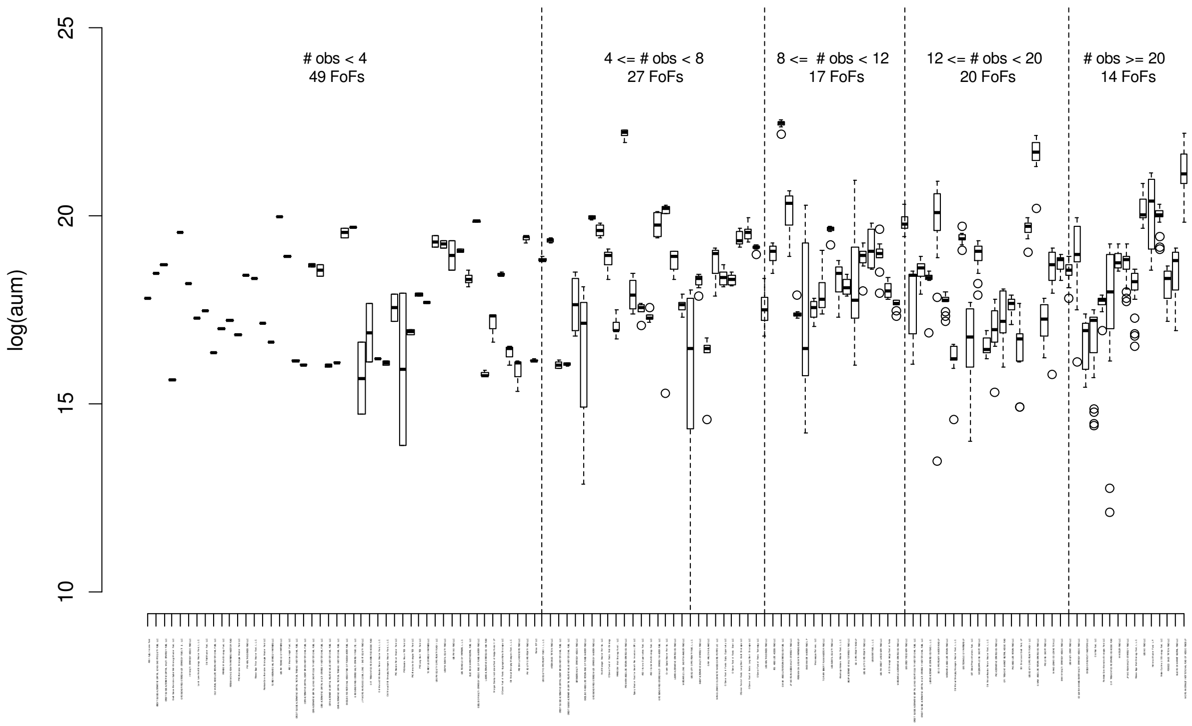


Figure 2

Assets under Management per FoF

The boxplots in this figure represent the distribution of $\log(\mathrm{AuM})$ for each of our 127 FoFs. Entries are sorted from left to right by the number of available fund observations. Printed at the top of each grouping are the number of quarterly observations available for – and the number of FoFs belonging to – that group.



Baseline Fixed-Effects Panel Regressions

- We start by running a fixed-effects (FE) panel regression model as suggested by our theoretical discussion in Section 3. Equation (9) presents our estimates for $i = 1,\ldots,127$ and for time t ranging from 2003Q1 to 2012Q4, where the statistical *t*-value is given in brackets:

$$\widehat{\log(n_{it})} = a_i + \underset{[12.843]}{0.473} \log(\mathrm{AuM}_{it}), \quad \bar{R}^2 = 0.85 . \tag{9}$$

- For this FE model the estimated slope is highly significant (*t*-value of 12.843) and, at 0.473, comes close to our prediction of 0.5. A Wald test for whether $\hat{b}$ is indeed insignificantly different from our prediction of 0.5 (with $H_0 : b = 0.5$) results in a $\chi^2(1)$-distributed test statistic of 0.54. The corresponding *p*-value of 0.45 does not allow us to reject the null hypothesis of $b = 0.5$.[12] Indeed FoF managers seem to set

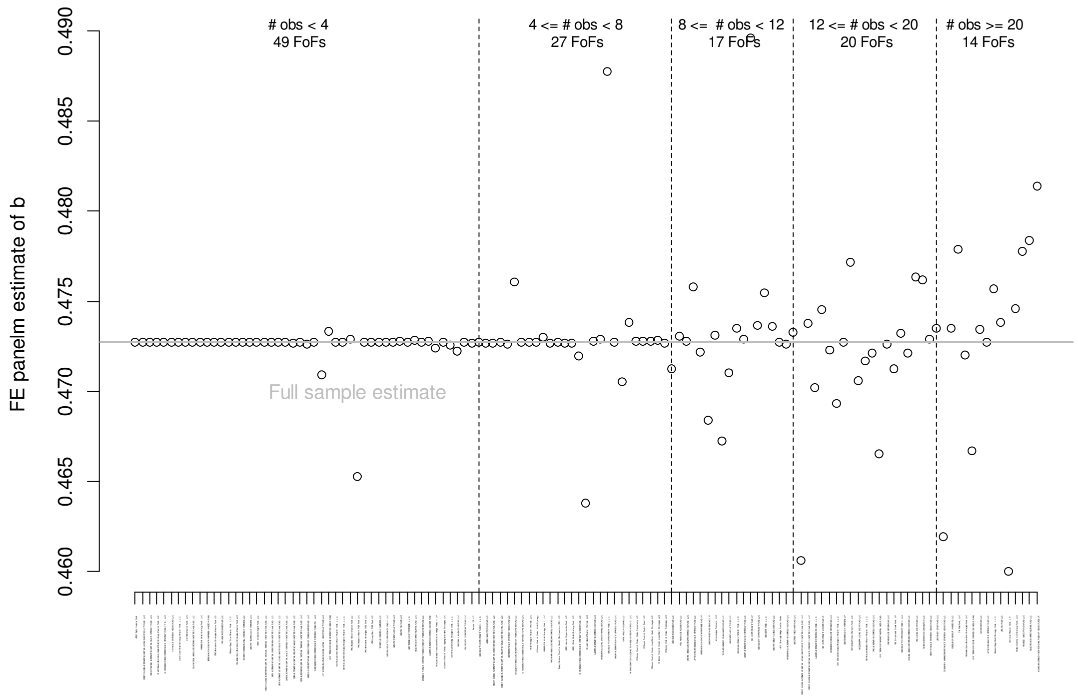
$$n^2 \propto aum . \tag{10}$$

[12] Including time effects does not change this result, although the estimated slope coefficient for the log of AuM does change marginally (to 0.503; *t*-value of 17.4). Yet testing against a slope of 0.5 results in a *p*-value of 0.97 – that is, making it even more difficult to reject our theoretical prediction. We also try omitting all funds with less than a year's worth of observations; although the slope estimate then drops to 0.45, it remains close (economically) to our conjectured value of 0.50.

Figure 3

Influential FoFs

This figure shows estimates for b in $\log(n_{it}) = a_i + b \cdot \log(\mathrm{AuM}_{it}) + \varepsilon_{it}$, where the $i\mathrm{th}$ FoF is dropped from the data set. Each data point reflects a regression beta estimated using 126 of 127 FoFs (i.e., with the $i\mathrm{th}$ FoF dropped). The gray line marks the full-sample estimate. Entries are sorted from left to right by the number of available fund observations. The top two printed lines give the number of quarterly observations available for a group of FoFs and the number of FoFs that belong to the indicated group.



Stacking versus Fixed Effects

- In order to validate our conjecture about the presence of individual effects in FoFs, we compare the FE regression with a pooled ordinary least-squares (OLS) regression. An absence of individual effects would imply that FoFs do not differ with respect to frictional costs, risk aversion (a reflection of different clienteles), or investment skill. The fit to this pooled OLS regression is given by

(11) $$\widehat{\log\left(n_{it}\right)} = \underset{[-12.448]}{-3.293} + \underset{[24.741]}{0.345} \log\left(\mathrm{AuM}_{it}\right), \quad \bar{R}^2 = 0.44 .$$

- Original data and fitted values for both regressions are plotted in Figure 5. We perform an F-test of fixed effects versus pooling by comparing the residual sum of squares for both models. With a test statistic of 20.14, the *p*-value is close to zero; hence we can reject the pooled OLS model. Pooling all observations amounts to ignoring individual effects, which will bias the estimated slopes (0.473 for FE regression versus 0.345 for pooled OLS regression).[13] This suggests that we should not ignore individual effects regarding to the FoF specific frictional costs, risk aversion, or investment skill.

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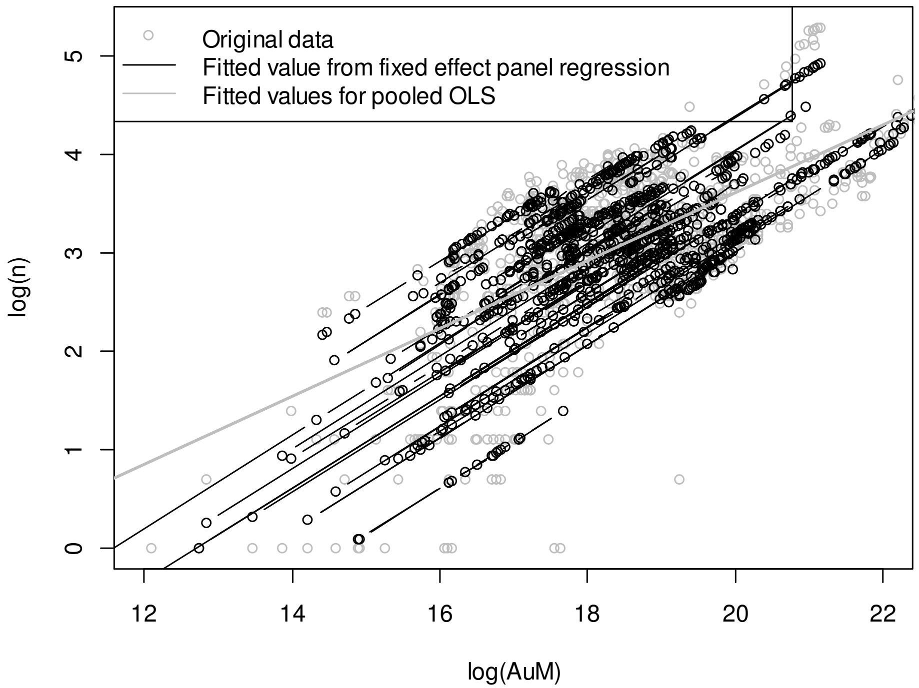
[13] We also compare the fixed-effects model with a random-effects (RE) model by means of the Hausmann test. The $\chi^2\left(1\right)$-distributed statistic takes a value of 21.27, so we can reject the null hypothesis of a RE panel model with high confidence (*p*-value of 0.000). Our slope parameter for the RE model is 0.47, which is both numerically and statistically close to our conjectured value of 0.5. A Wald test with $H_0 : b = 0.5$ results in a $\chi^2\left(1\right)$ of 2.26 (i.e., a *p*-value of 0.129).

Figure 4

Number of FoF Holdings versus Assets under Management

Each circle represents one pair of observations on $\log(\mathrm{AuM})$ and $\log(n)$ for our FoF data set. Fitted values for the FE panel regression $\widehat{\log\left(n_{it}\right)} = \hat{a}_i + \hat{b}\cdot\log\left(\mathrm{AuM}_{it}\right)$ and the pooled OLS regression $\widehat{\log\left(n_{it}\right)} = \hat{\alpha} + \hat{\beta}\cdot\log\left(\mathrm{AuM}_{it}\right)$ are marked by solid black and gray lines, respectively.



Break in Linearity?

- What would justify the break points in our model? When assets under management grow, an FoF can increase the number of its target funds as frictional diversification costs become proportionally smaller. However there may not exist enough target funds that satisfy this FoF investor's criteria. Is there a limit where the number of viable additional holdings is *not* increasing in assets under management?

- Following Hansen (1999), we estimate a threshold FE panel regression of the form

$$\log\left(n_{it}\right) = a_i + b \cdot \log\left(\text{AuM}_{it}\right) + \lambda \cdot \log\left(\text{AuM}_{it}\right) \cdot D_{it} + v_{it} \tag{12}$$

$$D_{it} = \begin{cases} 1 & \text{if AuM}_{it} \geq \gamma, \\ 0 & \text{otherwise.} \end{cases} \tag{13}$$

- Here γ is the threshold of AuM_{it} that activates an indicator variable, D_{it}. To estimate the unknown break point, we perform a grid search. The test statistic is bootstrapped under the null of no threshold effects.

- **We cannot reject the null hypothesis (of no break) at the 90% confidence level. FoFs seem to be still capable to hire (fire) individual funds that (do not) fulfill their selection criteria when their total assets under management increase (decline).**

Dynamic Panel Regressions

- We run a dynamic panel regression, where the dynamics are motivated by the partial adjustment hypothesis.

- While our model implicitly assumes that investors start a portfolio from cash rather than rebalancing from a set of current holdings, we relax this assumption and assume that the number of (equally weighted) assets evolve according to

$$(n_t - n_{t-1}) = (1-\theta)(n_t^* - n_{t-1}) \tag{14}$$

$$n_t^* = a + b \cdot aum_t + \varepsilon_t \tag{15}$$

$$n_t = a(1-\theta) + b(1-\theta)aum_t + \theta n_{t-1} + (1-\theta)\varepsilon_t \tag{16}$$

- Note, that (16) now contains a lagged dependent variable. This leads to biased estimates as residuals and regressors are now correlated

- We apply GMM with instrumental variables (lags of dependent variable) as suggested by Arellano/Bond (1991). Our new estimates become[15]

$$\hat{n}_t = \alpha_i + \underset{(8.28)}{0.58} \cdot aum_t - \underset{(2.50)}{0.30} \cdot n_{t-1}$$

- The lagged dependent variable turns out to be significant with a t-value of 2.5. A value of $\theta = 0.3$ means that while the adjustment is partial, 70% of the necessary adjustment is (on average) undertaken within one quarter. From this we can estimate the long run relationship between log(n) and log(aum):

(17)
$$\beta_{long-run} = \frac{\beta_{short-run}}{1+\theta} = \frac{0.58}{1+0.3} = 0.45$$

- Our dynamic panel regression model suggest that while the model holds in the long run, there can be some short term deviations from it.

- One weakness of this analysis is that we average across all funds. For example FoFs might take different adjustment decisions dependent on the liquidity terms of the individual funds they invest in. The next section will deal with this concern.

[15] As some of the fund data have gaps and or few observations, entire FoF's drop out of the estimation and the estimation universe relative to a static regression becomes different.

A Closer Look at Holdings Data

- While our data set consists of holdings level data, we so far only used this data to calculate n. In this section, we provide a deeper look at the data to see whether this “micro” view is consistent with the previous “macro” view. In short: do individual FoF’s display equal weighting of their constituent funds?

- First, we plot the average (log) number of funds versus the average (log) number of assets under management and simultaneously display the amount of active money defined as

(18) $$active - money = \frac{1}{2}\sum_{i=1}^{n}\left|w_i - \frac{1}{n}\right|$$

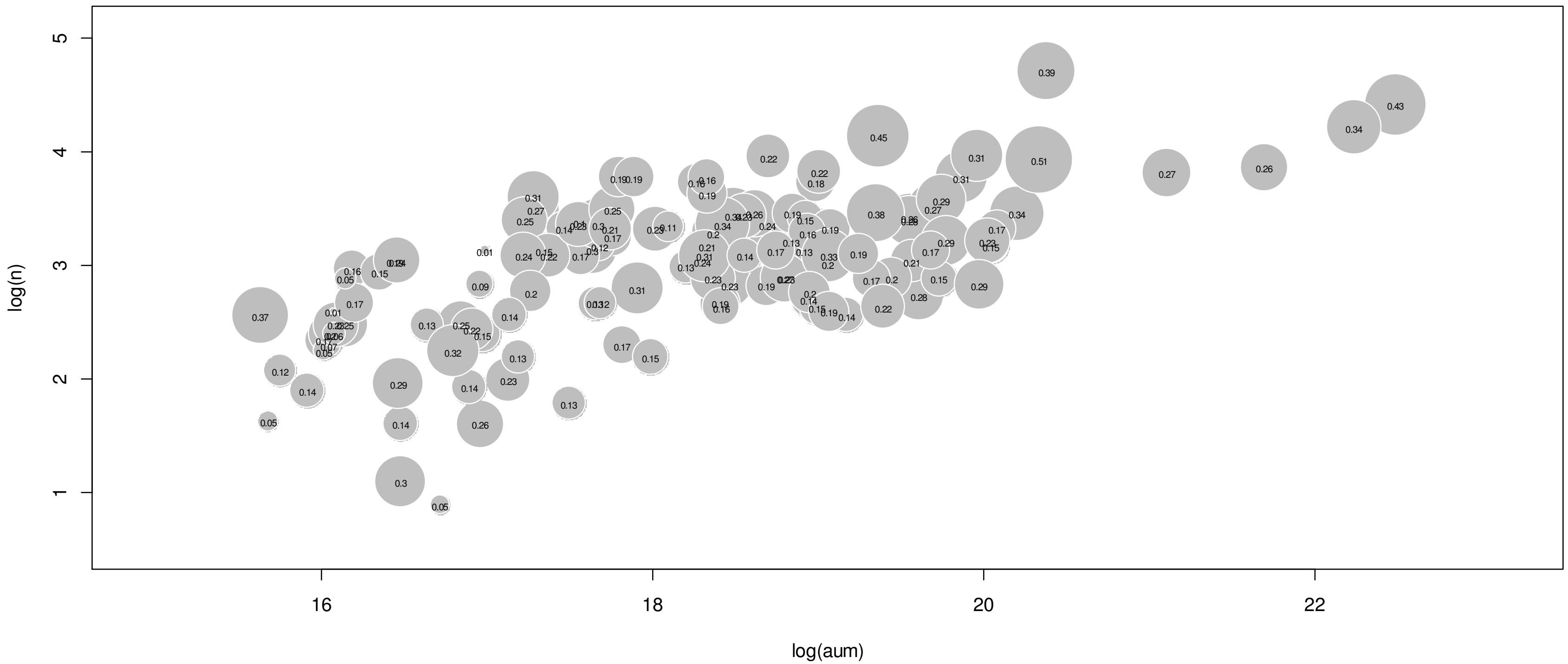
Equation (18) defines the fraction of assets under management that need to be traded, to arrive at an equally weighted portfolio. It is plotted for all FoF’s in Figure 5.

- What can explain deviations from 1/n even if investors still apply 1/n?
    - Equal risk weightings
    - Weight drift in the presence of target fund illiquidity

Figure 5

Active Money

For each fund we plot the average (log) number of funds versus the average (log) number of assets under management. We also display the amount of active money (i.e. the fraction of assets under management that needs to be traded to arrive at an equally weighted portfolio) as the size of the corresponding bubble.



- Note, the amount of active money is rarely above 40%. Is this a high or low number? When do we call deviations from 1/n inconsistent with 1/n?

- Equal risk instead of 1/n. If investors apply volatility weighting they would allocate equally to identical risk streams. Our model of naïve diversification under frictional costs remains unaffected. Only the definition of what constitutes a base asset has changed now.

- How much active money can we attribute to this effect? To get an idea we will use the EDHEC data on hedge fund style indices and calculate the amount of active money that would arise from equal risk weightings. We use monthly data for the period January 1997 to June 2014 to calculate the volatility of each hedge fund style (leaving out fund of funds) from

(19)
$$E\left(active - money\right)_{risk-scaling} = \frac{1}{2}\sum_{i=1}^{12}\left|\underbrace{\left(\frac{\frac{1}{\sigma_i}}{\sum_{i=1}^{12}\frac{1}{\sigma_i}}\right)}_{\substack{risk-parity\\ weight}} - \underbrace{\frac{1}{12}}_{\substack{equal\\ weight}}\right| = 0.17$$

- In other words 17% active money arises from risk scaling (1/n investing for risk equalized assets) alone, i.e. even if fund of funds are conceptually wedded to applying the idea of equal weighting of (now of risk streams)

- Second, active money will also be a function of holding period (driven by liquidity or institutional constraints) and cross sectional volatility of funds returns. Let us ask, how far can we expect w_i drift by pure chance? What value can $E(active - money)$ take? Note, that individual fund weights evolve according to relative portfolio performance, where r_p denotes the FoF's performance

(20) $$w_i^i = \left(\tfrac{1}{n}\right)\left(1 + r_i\right)\left(1 + r_p\right)^{-1}$$

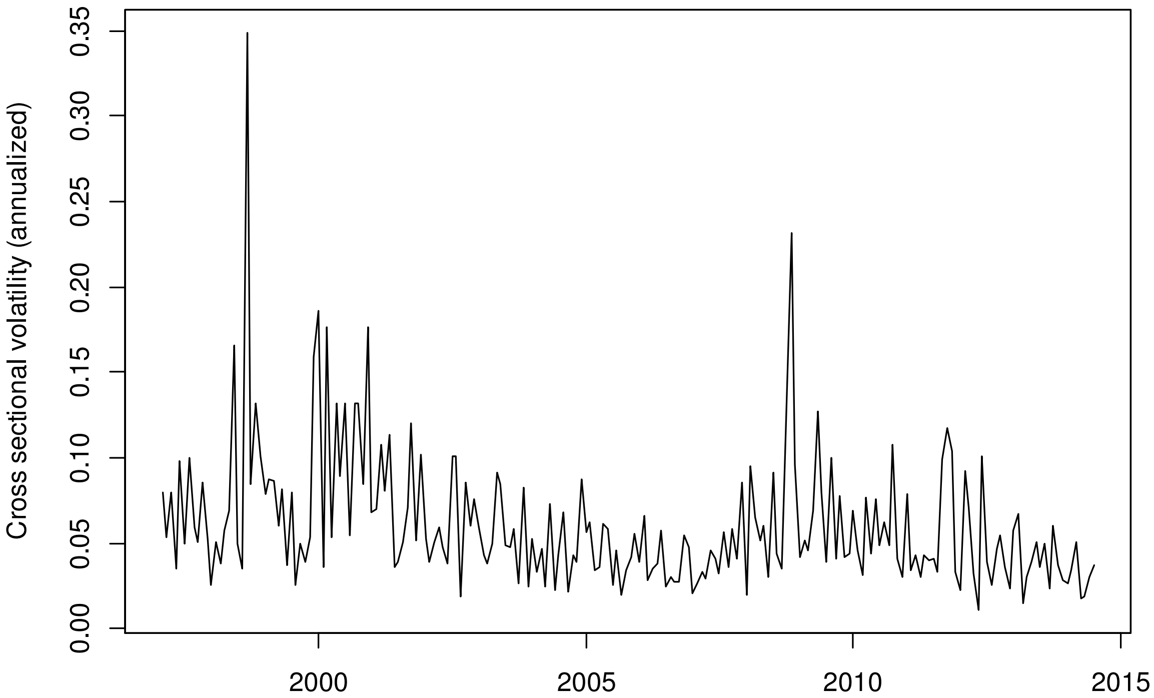
- Substituting (25) into (18) and taking expectations (using the truncated normal distribution) we arrive at the formula below, where the second term in brackets denotes the cross sectional volatility (responsible for weight diversion) and Δt the investment horizon in which portfolio holdings are left untouched (i.e. the time for weights to drift apart from equal weighting).

(21) $$E\left(active - money\right)_{drift} \approx \left(\sqrt{\tfrac{1}{2\pi}}\right)\left(\sqrt{\tfrac{1}{n}\sum_{i=1}^{n}\left(r_i - r_p\right)^2}\right)\sqrt{\Delta t}$$

Figure 6

Cross Sectional Volatility of Hedge Funds

We plot the cross sectional volatility of EDHEC indices calculated from monthly data for the period January 1997 to June 2014. Fund of fund returns are removed (as they are already a linear combination of existing styles and bias cross sectional volatility downwards).



- In line with industrial practice we assume that individual mandates stay for a 3 year period. This reflects liquidity limitations as well as the client's intent not to overreact on short-term noise.

- We estimate the time series of cross sectional volatility of hedge fund styles (plotted in) from the EDHEC hedge fund indices. For our calculations we use the series mean of 22.4%. However, individual funds will display larger dispersion than their industry aggregates. We therefore allow a 50% higher cross sectional volatility for individual funds than the historical average for hedge fund style indices. We arrive at

$$E\left(active - money\right)_{drift} \approx \left(\sqrt{\frac{1}{2\pi}}\right)\left(0.22 \cdot 1.5\right)\sqrt{3} = 22\%$$

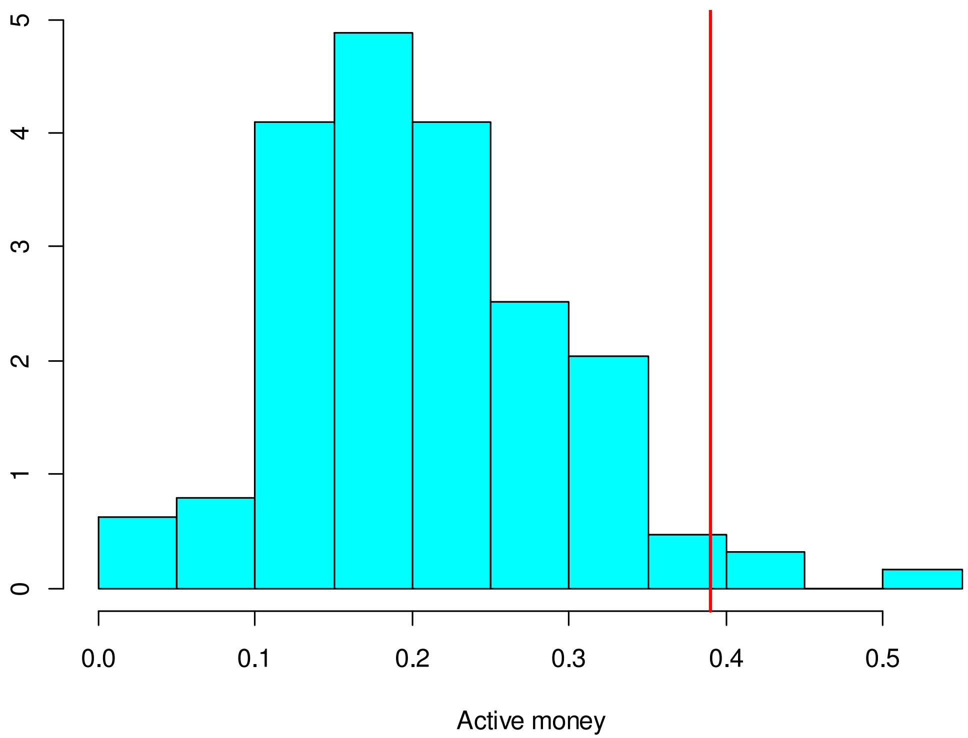
- In combination, this leaves us with an amount of active money of 39%

- This is still consistent with the idea of equal weighting (of risk equalized assets). In our data set only 3% of all FoF's display active money larger than 39%. We therefore find, that the initial conjecture of FoFs following the idea of 1/n, is well supported by our data set.

Figure 7

Distribution of Active Money

We plot the histogram of active average active money per FoF together with the amount of active money (red vertical line) we would expect simply as a consequence of risk scaling and weight drift.



Conclusion

- Diversification is no free lunch. It requires size (assets under management) to offer diversification due to frictional diversification costs (due diligence costs)

- We find a positive log linear relation between the number of funds, n, and the assets under management for (hedge) fund of funds, aum:

$$n^2 \propto aum.$$

- The presented evidence is econometrically robust and consistent with a model of naive diversification under frictional diversification costs.

- Actual holdings of fund of funds are consistent with naïve diversification

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