Asset Pricing with Endogenous β
QMI Project Summary

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Introduction

The main workhorse of the empirical asset pricing research as well as much of the investment industry practices, in search of efficient benchmarks, is the factor structure of the relation between expected returns and risk. The theoretically appealing CAPM market equilibrium model (Sharpe, 1964; Lintner, 1965) has found very little empirical support. Early evidence showed that the empirical Security Market Line–the linear relation between expected returns and the β, the exposure to the single market risk–is way flatter than the theory predicts. The failure to uncover supporting evidence was initially thought to be a failure of observing the true market risk (Roll, 1977) and, hence, the inability to measure the market β's correctly. The market β was later declared "dead" (Fama and French, 1992, 1993, 1996) given strong empirical evidence that β has no predictability power in explaining the cross-section of expected returns. Recent evidence has shown that the β "puzzle" goes further in that the β arbitrage (Frazzini and Pedersen, 2014), i.e. betting against β, is a profitable strategy.

One weakness of the traditional CAPM model is that the stock return volatilities and correlations and, hence, the β's are exogenous. This assumption, however, is not innocuous for a number of reasons: (i) individual stock beta is measured with error, even if assumed constant, (ii) the beta's along with the second moments change over time and (iii) it is unclear what is the source of the stock return volatility. The original volatility puzzle (Shiller, 1981; Leroy and Porter, 1981), stemmed from the observation that the fluctuations in returns are larger than the fluctuations in cash-flows. Several resolutions of this puzzle have been proposed based on

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fluctuations in risk-aversion (Campbell and Cochrane, 1999), small persistent fluctuations in expected cash-flow growth (Bansal and Yaron, 2004), fluctuations in the probability of rare disasters (Barro, 2006; Gabaix, 2012; Wachter, 2013), and others. The debate as to which factor is the principal driver of stock price fluctuations is still open. Regardless, there has been little research–see for example Da (2009) and Da and Warachka (2009)–, as to the implications of these failures on the cross-section of β's which is the aim of this study.

Summary

We develop an equilibrium model of asset returns for the cross section with endogenous exposure to the priced risk(s) that is consistent with both the equity premium and the stock return volatility. Essentially, we use time-series properties of aggregate stock returns, the risk-free rate and potentially returns on other large asset classes to construct a stochastic discount factor and then focus on the cash-flows of individual stocks and stock portfolios.

The strategy of this investigation is inspired by the work of Menzly, Santos and Veronesi (2004), Cochrane, Longstaff and Santa-Clara (2007) and Santos and Veronesi (2010) and is based on a novel approach in modelling cash-flows for the cross-section. Specifically, we model the cash-flows of different assets or portfolios of assets, e.g. industries, as fractions of the aggregate stock market dividend. Our choice of the portfolios is crucial because of the necessary assumption that the *dividend shares* are jointly stationary. In the same spirit, we also model the aggregate stock market dividend as a fraction of the aggregate consumption and take consumption as exogenous. The aggregate consumption is modelled as conditionally normal with time-varying moments.

The dividend shares are denoted by δ_t^i. The modelling technique, which employs some form of orthogonalization of the dividend share processes allows for both closed form analytics as well as it facilitates model estimation in a straight forward way. We assume that there is a set of $n-1$ factors that drive jointly the cash-flow shares of the n assets. The result is that the dividend share of an asset or portfolio follows a Wright-Fisher process, as advocated by Cochrane, Longstaff and Santa-Clara (2007),

$$d\delta_t^i = \phi_i\left(\bar{\delta}_i - \delta_t^i\right) + \sqrt{(1-\delta_t^i)\delta_t^i}\left(\eta_i dW_t^m + \sigma_i dW_t^i\right) \tag{1}$$

where W^i and W^m are independent Brownian motions with W^m representing the priced risk and W^i being the idiosyncratic cash-flow risk. By the Fundamental Theorem of Asset Pricing, the market is arbitrage free if there exist an unique stochastic discount factor process Λ_t such that the price of asset i is given by

$$P_t^i = \mathbb{E}_t \int_t^\infty \frac{\Lambda_\tau}{\Lambda_t} D_\tau^i d\tau \tag{2}$$

for all assets, where D_t^i denotes the cash-flow of asset i at time t. The second main element of the model is the specification of the stochastic discount factor Λ. Instead of being limited by specific preferences, we assume that the stochastic discount factor follows a Geometric Brownian Motion with time-varying risk-free rate and price of risk, which will be estimated and filtered using aggregate asset price data. We assume that there is only one priced risk for two reasons. Firstly, theoretically it is hard to justify factors other than wealth driving the marginal utility of consumption. Secondly, the aim is to see how far we can get with a one factor model in explaining the cross-section.

The first implication of the model is that when shocks to the dividend shares are independent, that is η_i is zero, then on average the β of the particular asset is 1. It fluctuates around 1 depending on whether the dividend share is above or below its long run mean. This result already creates a strong prior for the true exposure of individual stocks or portfolios. Once we assume a value for η_i different than one, the average β then differs from one, as expected. It is possible that the priced risk has a significant role in driving dividend shares, which makes it important to investigate how large can the variation in beta's be, unconditionally. Specifically, we will perform a sensitivity analysis on the parameters of the model to investigate how the beta of an asset varies with the parameters as well as the state of the dividend share.

Given a parameterized economy, the conditional beta of an asset depends on the parameters driving the dividend share and the state of the dividend share, especially in relation to its long-run mean. However, since for certain parameter sets the main driving factor is the δ_t^i, a simple empirical strategy that does not require any estimation is to compare the value of the dividend shares with their historical averages–a proxy for their long-run means–and test whether this statistic can predict expected excess returns over the cross section. We will also perform an in-sample analysis whereby we will determine the state of the dividend share using the entire sample to determine the long-run means.

The final part of the analysis will be the estimation of the dividend share processes for a set of portfolios for which the dividend shares are expected to be stable, as for example the industry portfolios. This will allow us to estimate both the unconditional as well as the conditional β's for all portfolios in the analysis and use them to predict expected returns. For the estimated processes we will be able to compare the properties of the empirical asset returns with those predicted by the model. Then, we can compare the predicted expected returns with the realized.

The estimation of the dividend share processes will be done in two ways. Firstly, we will use purely cash-flow data, either dividends, adjusted for share repurchases, or earnings. However, given the small amount of this data and the fact that they are not forward looking we will also employ a second method. Namely, we will use the model to draw the connection between stock returns and shocks to the dividend shares. Specifically, according to the model, stock returns are driven by shocks to the price of risk, the risk-free rate, the moments of the growth rate of the aggregate economy and the dividend shares. The latter approach will allow us to estimate the processes with high accuracy, it will be applicable at both the individual stock level as well as at a portfolio level and it will be consistent with the historical stock returns.

Concluding remarks

We believe that this investigation will shed a different light on what drives returns at the cross-section and inform us whether we need to investigate further the cash-flow characteristics of the assets. This would give a different direction of investigation from the approach that searches for purely return based factors.

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